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Joint effect of stock threshold level and production policy on an unreliable production environment

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ABSTRACT

In this article we develop an economic manufacturing quantity (EMQ) model subject to stochastic machine breakdown, repair and stock threshold level (STL). Instead of constant production rate, in this model production rate is considered as a decision variable. Since, the stress of the machine depends on the production rate, failure rate of the machine will be a function of the production rate. Again, in this article consideration of safety stock in all existing literature is replaced by the concept of stock threshold level (STL). Further, extra capacity of the machine is considered to buffer against the possible uncertainties of the production process where machine capacity is predetermined. The basic model is developed under general failure and general repair time distributions. Since, the assumption of variable production rate makes the objective function quite complex, so main emphasis is given on computational methodology to solve the present problem. We suggest two computational algorithms for the determination of production rate and stock threshold level which minimize the expected cost rate in the steady state. Finally, through numerical examples we illustrate the key insights of our model from managerial point of view.

1. Introduction

In this age of technology, modern manufacturing systems are becoming more and more complex. Even though they are more reliable than their predecessor, they are still subject to deterioration and failure with usage and age. Deterioration causes lower production rate and lower production quality. So, production, quality and maintenance are three important aspects in any manufacturing firm. Since global business competition is increasing day-by-day, managers in manufacturing industries are facing great challenges every day to produce better quality product and to provide better customer services than before. The classical EMQ model [1] usually does not take into account the process deterioration and machine breakdown during production run. But these models rarely meet the practical situations. Over the decades, numerous research efforts have been undertaken to fit closely to the real world situations by incorporating the imperfections of the production process (*i.e.*, quality and yield issues) and equipment (*i.e.*, machine breakdown and repair) in the classical lot sizing decisions. Mainly, imperfect EMQ models are extended in two parallel directions. In one direction it is assumed that the process may shift from 'in-control' state to 'out-of-control' state due to usage and age and as a consequence, begins to produce some percentage of defective items. Initial work was done simultaneously by Porteus [2] and Rosenblatt and Lee [3]. Porteus [2] developed a model where the process may shift from 'in-control' state to 'out-of-control' state with a given probability each

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Table 1
A comparison study of our present model with the related models.

Author	Model type	Production rate	Stock threshold level	Extra capacity of the machine	Failure rate pattern
Cheung and Hausman [20]	Imperfect EMQ model with maintenance and safety stocks	Fixed and production rate equals to demand rate	N	Y	Independent of production rate
Dohi et al. [21]	Extension of the above [20] model	Fixed and production rate equals to demand rate	N	Y	Independent of production rate
Our model	Imperfect EMQ model *i*th maintenance and stock threshold level	Variable and production rate greater than demand rate	Y	Y	Depends on production rate

time it produces an item. In a similar type of works, assuming exponential process shift distribution Rosenblatt and Lee [3] concluded that optimal production run length is shorter than that of the classical EMQ model. After that, several researchers [4–11] devoted their time to extend the classical EMQ model in this particular directions.

In the other direction of research for deteriorating production system, during production run the process or machine may breakdown at any random time. After machine failure, a repair action is carried out immediately. The time to repair the machine may be fixed or random. Shortages may arise due to longer repair time.[1] Shortages may be of lostsales[2] or backlogged[3] type after the resumption of the machine. Again, if there is no machine failure during the production run, then a maintenance action is carried out at the end of the production to bring back the system to initial working condition before the start of the next production run. Sometimes to increase the system reliability or to delay the occurrence of machine failure, maintenance actions are considered during the production run. The initial work was done by Groenevelt et al. [12] who studied the impact of machine breakdown and repair on the optimal lot sizing decisions. Assuming exponential inter failure time distribution and instantaneous repair time they showed that the optimal lot size is always larger than that of the classical EMQ model and always increases with the failure rate. They justified their conclusions by the argument that larger production lot size is in order to compensate the production loss due to machine breakdown. They [13] further extended their model [12] by incorporating the issue of safety stock required to meet the managerial prescribed service level. Kim et al. [14] reformulated Groenevelt et al. [12] model by assuming constant failure rate and concluded that the optimal lot size does not increase always with the increase of failure rate which contradicts Groenevelt et al.'s claim. Recently, Chakraborty et al. [15,16] developed models by integrating the joint effect of process deterioration and machine breakdown by assuming general process shift, machine breakdown, corrective repair and preventive maintenance time distributions.

In all the above mentioned extended EMQ models, production rate is considered as predetermined and inflexible. Khouja & Meherez [17] first incorporated the concept of variable production rate into the EMQ literature. They formulated their model under exponential process shift distribution with mean as an increasing function of the production rate. As a random nature of the production system, safety stock play an important role to guarantee the continuous delivery of the product during the interruption of the production due to machine breakdown.Then Giri et al. [18,19] extended EMQ models with this assumption of variable production rate. Groenevelt et al. [13] first considered the impact of safety stock into the machine breakdown model. Later Cheung and Hausman [20] investigated the joint effect of preventive maintenance and safety stocks on unreliable production system considering extra capacity of the machine. In many manufacturing system extra capacity of the machine is maintained to buffer against various uncertainties of the production system. Dohi et al. [21] reconsidered the Cheung and Hausman [20]'s model from theoretical point of view. Recently, Chakraborty and Giri [22] developed an imperfect EMQ model and studied the combined effect of process deterioration, machine breakdown, corrective and preventive maintenance together with the impact of buffer stocks on the optimal decisions for an unreliable production system.

In the above existing literature where extra capacity of the machine and safety stock were considered, the production rate was assumed to be constant. Further, in contrast to the existing literature, in our present paper instead of maintaining the safety stock at the beginning of each production cycle we have considered a stock threshold level (STL). This STL is considered as a decision variable. Thus the purpose of the present work is to study the impact of the joint implementation of machine breakdown, maintenance and stock threshold level (STL) on a stochastic EMQ model where failure rate is assumed to be a function of the production rate. Here, we have considered the extra capacity of the machine. Instead of constant production rate, in this model the production rate is considered as a decision variable. A comparison study of our present model with the related existing literature is given in Table 1. We organize our paper in the following manner. The following section describes the assumptions and notation used through out the paper. The model description is given in Section 3. In Section 4, the model is formulated under general failure and general repair time distribution. Some solution approaches to obtain the optimal solution of our model are described in Section 5 through algorithms. Section 6 proposed some properties of the

[1] This case may arise for random repair time.
[2] In case of lostsales, shortages are not delivered after resumption of the production. They are totally lost. In this case some goodwill loss costs are involved into the expected cost of the system.
[3] In backlogged case after the resumption of the machine shortages are met first either totally or partially.