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# Stability and local bifurcation analysis of functionally graded material plate under transversal and in-plane excitations ☆

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ABSTRACT

In this paper, stability and local bifurcation behaviors for a simply supported functionally graded material (FGM) rectangular plate subjected to the transversal and in-plane excitations in the uniform thermal environment are investigated using both analytical and numerical methods. Three kinds of degenerated equilibrium points of the bifurcation response equations are considered, which are characterized by a double zero eigenvalues, a simple zero and a pair of pure imaginary eigenvalues as well as two pairs of pure imaginary eigenvalues in nonresonant case, respectively. With the aid of Maple and normal form theory, the explicit expressions of transition curves are obtained, which may lead to static bifurcation, Hopf bifurcation and 2-D torus bifurcation. Finally, the numerical solutions obtained by using fourth-order Runge–Kutta method agree with the analytic predictions.



## 1. Introduction

With the rapid development of technology in many fields such as aircraft, space shuttle, automobile etc., many new materials possessing excellent and special properties occur. The functionally graded material (FGM) is one of the extremely excellent materials, which is inhomogeneous composite material whose property varies continuously and smoothly from a ceramic surface to a metallic surface in a specified direction of the structure. The ceramic surface protects the metallic surface from corrosion, whereas the metallic part offers strength and stiffness to the structure. Because of the special component, corrosion is decreased.

Recently, researchers have done various studies on the FGM in order to use it effectively and efficiently. In [1], Hao et al. investigated the FGM plate in the uniform thermal environment and revealed that chaotic motions exist under certain conditions. Shen et al. studied the nonlinear vibration of the FGM plate in [2,3] and considered the free vibration and parametric resonance of shear deformation FGM cylindrical panels in [4]. Hu and Zhang [5] investigated the dynamical behaviors of thin circular functionally graded plate subjected to one-term and two-term transversal excitations in some thermal environment. Piovan and Sampaio [6] considered the vibrations of axially moving flexible beams composed of functionally graded materials.

With the application of the intrinsic harmonic balancing [7] and unification technique [8,9], Yu and Huseyin studied the stability and bifurcation behaviors of the systems which characterized by a double-zero eigenvalues [8], a simple zero and a

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pair of pure imaginary eigenvalues [9], two distinct pairs of pure imaginary eigenvalues [10] and so on [11,12]. In paper [13–18], Yu et al. obtained the corresponding normal forms with Maple.

In this paper, we consider local dynamic behaviors for the FGM rectangular plate in the case of 1:1 internal resonance and primary resonance using both analytical and numerical methods. Three types of degenerated equilibrium points will be studied in detail, which are characterized by a double zero eigenvalues, a simple zero and a pair of pure imaginary eigenvalues and two pairs of pure imaginary eigenvalues in nonresonant case, respectively. The stability conditions and transition curves leading to static bifurcation, Hopf bifurcation and 2-D torus bifurcation are obtained with the aid of normal form theory, bifurcation and stability theory. All numerical results agree with the analytic predictions.

This paper is organized as follows: in Section 2, the averaged equations of transverse motion of the FGM plate are given and the stability conditions of initial equilibrium solution are obtained explicitly. The bifurcation behaviors of the system in the vicinity of the three types of degenerated equilibrium points are presented in Section 3, which is followed in Section 4 by a short summary.

## 2. Formulation of the problem

This paper focuses on the stability and bifurcation behaviors of a functionally graded material rectangular plate subjected to the in-plane and transversal excitations in the uniform thermal environment. The model is shown in Fig. 1. The edge width, length and thickness of the FGM rectangular plate in the $x$, $y$ and $z$ directions are $a$, $b$ and $h$ respectively. Especially the width-to-length ratio of the FGM rectangular plate is $a/b = 1$. The transversal excitation subject to the FGM plate is of the form $F(x,y)\cos\Omega_1 t$ and the in-plane excitation along $x$ direction at $x = 0$ and $x = a$ is given by $p = p_0 - p_1\cos\Omega_2 t$. Here, $\Omega_1$ and $\Omega_2$ are the frequencies of the transversal excitation and in-plane excitation, respectively.

In [1], the nonlinear dimensionless governing differential equations of transverse motion of the FGM rectangular plate are obtained as follows,

$$\ddot{w}_1 + \omega_1^2 w_1 + a_1\dot{w}_1 + a_2 w_1 cos\Omega_2 t + a_3 w_1^2 + a_4 w_2^2 + a_5 w_1 w_2^2 + a_6 w_1^3 + a_7 w_1 w_2 = f_1 cos\Omega_1 t, \ddot{w}_2 + \omega_2^2 w_2 + b_1\dot{w}_2 + b_2 w_2 cos\Omega_2 t + b_3 w_1 w_2 + b_4 w_1^2 + b_5 w_2^2 + b_6 w_2 w_1^2 + b_7 w_2^3 = f_2 cos\Omega_1 t.$$

where $\omega_1$ and $\omega_2$ are the amplitudes of two modes and all the coefficients can be found in [1]. With the asymptotic perturbation method [19,20], Hao et al. transformed the aforementioned equations into the averaged equations as follows,

$$\dot{x}_1 = \mu_1 x_1 + (\sigma_1 + \alpha)x_2 + \gamma_1 x_4 + Nf_1^1, \tag{1a}$$

$$\dot{x}_2 = (-\sigma_1 + \alpha)x_1 + \mu_1 x_2 + \gamma_1 x_3 + Nf_2^1, \tag{1b}$$

$$\dot{x}_3 = \gamma_2 x_2 + \mu_2 x_3 + (\sigma_2 + \beta)x_4 + Nf_3^1, \tag{1c}$$

$$\dot{x}_4 = \gamma_2 x_1 + (-\sigma_2 + \beta)x_3 + \mu_2 x_4 + Nf_4^1, \tag{1d}$$

where $\alpha = \alpha_1 + \alpha_2 f_1 + \alpha_{11} f_2,\ \beta = \beta_1 + \beta_2 f_1 + \beta_3 f_2,\ \gamma_1 = \alpha_3 f_2 + \alpha_{11} f_1,\ \gamma_2 = \beta_4 f_2 + \beta_5 f_1$ and $\alpha_i\ (i = 1,2,3,11)$ and $\beta_i\ (i = 1,\ldots,5)$ can be found in [1].The nonlinear functions $Nf_i^1\ (i = 1,\ldots,4)$ are presented in Appendix A. Especially, $\mu_i\ (i = 1,2)$ are two combined parameters, including damping parameters. The resonant relations are $\omega_1 = \frac{\Omega_1}{2} + \epsilon^2\sigma_1,\ \omega_2 = \frac{\Omega_1}{2} + \epsilon^2\sigma_2,\ \Omega_1 = \Omega_2 = \Omega$, where $\sigma_i\ (i = 1,2)$ are detuning parameters.

The Jacobi matrix of (1) evaluated at the initial equilibrium solution $(x_1, x_2, x_3, x_4) = (0,0,0,0)$ is as follows:

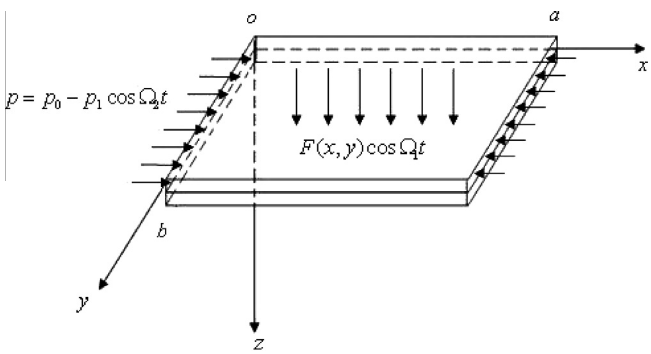


**Fig. 1.** The model of a FGM rectangular plate and the coordinate system.