Applied Mathematical Modelling 37 (2013) 6909–6917

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A method based on similarity measures for interactive group decision-making with intuitionistic fuzzy preference relations

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ARTICLE INFO

Article history:
Received 19 March 2012
Received in revised form 20 December 2012
Accepted 8 January 2013
Available online 5 March 2013

Keywords:
Intuitionistic fuzzy preference relations
Group decision-making problem
Similarity measures
Consensus
Interaction

ABSTRACT

In this paper, we study the group decision-making problem in which the preference information given by experts takes the form of intuitionistic fuzzy preference relations, and the information about experts' weights is completely unknown. We first utilize the intuitionistic fuzzy weighted averaging operator to aggregate all individual intuitionistic fuzzy preference relations into a collective intuitionistic fuzzy preference relation. Then, based on the degree of similarity between the individual intuitionistic fuzzy preference relations and the collective one, we develop an approach to determine the experts' weights. Furthermore, based on intuitionistic fuzzy preference relations, a practical interactive procedure for group decision-making is proposed, in which the similarity measures between the collective preference relation and intuitionistic fuzzy ideal solution are used to rank the given alternatives. Finally, an illustrative numerical example is given to verify the developed approach.

1. Introduction

The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] to generalize the concept of Zadeh's fuzzy set [2]. Each element in an IFS is expressed by an ordered pair, and each ordered pair is characterized by a membership degree and a non-membership degree. The sum of the membership degree and the non-membership degree of each ordered pair is less than or equal to one. Since it was first introduced in 1986, the IFS theory has been widely investigated and applied to a variety of fields [3–17]. Similarity measures and distance measures between intuitionistic fuzzy sets (IFSs), as an interesting and important research topic in IFS theory, have been receiving more and more attention in recent years. Szmidt and Kacprzyk [18] defined four basic distances between IFSs: the Hamming distance, the normalized Hamming distance, the Euclidean distance, and the normalized Euclidean distance. Li and Cheng [19] also proposed similarity measures of IFSs and applied these measures to pattern recognition. But Liang and Shi [20], Mitchell [21] pointed out that Li and Cheng's measures are not always effective in some cases, and made some modifications respectively. Grzegorzewski [22] proposed some distance measures between IFSs and/or interval-valued fuzzy sets based on the Hausdorff metric, which are the generalizations of the well known Hamming distance, the Euclidean distance and their normalized counterparts. However, all the above similarity measures consider just a distance between the IFSs that are being compared, and can not answer the question if the compared IFSs are more similar or more dissimilar. To overcome this drawback, Szmidt and Kacprzyk [23] proposed a similarity measure, which involves both similarity and dissimilarity between IFSs, and showed its usefulness

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http://dx.doi.org/10.1016/j.apm.2013.01.044

in medical diagnostic reasoning. But Xu and Yager [24] pointed out that Szmidt and Kacprzyk's method is inconvenient in the practical applications because it is somewhat difficult to express enormous numbers. By analyzing Szmidt and Kacprzyk's results, they extended the idea of the TOPSIS of Hwang and Yoon [25] to develop an improved similarity measure between two IFSs, and applied it for consensus analysis in group decision-making based on intuitionistic fuzzy preference relations.

Preference relations form a useful tool in expressing decision makers' preferences over alternatives. Consider that, in some real-life situations, a decision maker (DM, member, expert,etc.) may not be able to accurately express his/her preferences for alternatives due to that (1) the DM may not possess a precise or sufficient level of knowledge of the problem; (2) the DM is unable to discriminate explicitly the degree to which one alternative are better than others [26], in such cases, the DM may provide his/her preferences for alternatives to a certain degree, but it is possible that he/she is not so sure about it [27]. Thus, it is very suitable to express the DM's preference values with the use of intuitionistic fuzzy values rather than exact numerical values or linguistic variables [28–36]. Szmidt and Kacprzyk [37] generalized the fuzzy preference relation to the intuitionistic fuzzy preference relation, and defined the concepts of intuitionistic fuzzy core and consensus winner. Xu [38] investigated the properties of intuitionistic fuzzy preference relations by constructing the score matrix and accuracy matrix, and he also gave the research of the group decision method with the intuitionistic fuzzy preference relations. Xu [39] developed a method for estimating criteria weights from intuitionistic preference relations. Xu and Yager [24] established a consensus reaching process of group decision-making with intuitionistic fuzzy preference relations based on similarity measures. Gong [40] proposed the least squares and the goal programming model to derive the priority vector of the intuitionistic preference relations. Gong et al. [41] investigated additive consistent properties of the intuitionistic fuzzy preference relation. Usually, the DMs come from various research domains or have different knowledge backgrounds, and they necessitate different weights in deciding group preferences. Thus, how to estimate DMs' weights from intuitionistic preference relations is an interesting and important issue, which no investigation has been devoted to. In this paper, we shall develop a method based on similarity measures for estimating DMs' weights from intuitionistic preference relations. We first utilize the intuitionistic fuzzy weighted averaging operator to aggregate all individual intuitionistic fuzzy preference relations into a collective intuitionistic fuzzy preference relation. Then, we develop an approach to derive the experts' weights directly from the degree of similarity between the individual intuitionistic fuzzy preference relations and the collective one. Furthermore, we present a practical interactive procedure for group decision-making with intuitionistic fuzzy preference relations, and develop a method based on similarity measures to rank and select the alternatives.

In order to do so, the remainder of this paper is structured as follows. Section 2 reviews some basic concepts and similarity measures related to intuitionistic fuzzy sets. Section 3 develops an approach to interactive group decision-making with intuitionistic fuzzy preference relations. Section 4 provides a practical example to illustrate the developed approaches, and Section 5 concludes the paper.

2. Preliminaries

Let a set X be fixed, an intuitionistic fuzzy set A in X is given by Atanassov [1] as an object having the following form:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\} \tag{1}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $v_A(x) : X \to [0,1]$ determine the degree of membership and the degree of nonmembership of the element $x \in X$, such that $0 \leqslant \mu_A(x) + v_A(x) \leqslant 1$ for all $x \in X$. In addition $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy of x to A, or called the degree of hesitancy of x to A. Especially, if $\pi_A(x) = 0$, for all $x \in X$, then the IFS A is reduced to a fuzzy set. The complement of A is given by Szmidt and Kacprzyk [23]: $A^c = \{\langle x, v_A(x), \mu_A(x)\rangle | x \in X\}$.

Clearly, a prominent characteristic of IFS is that it assigns to each element a membership degree, a nonmembership degree and a hesitation degree, and thus, IFS constitutes an extension of Zadeh's fuzzy set [2] which only assigns to each element a membership degree.

Xu and Yager [15] called each triple $(\mu_A(x), v_A(x), \pi(x))$ as intuitionistic fuzzy value (IFV), and for convenience, denoted an IFV by $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$, where

$$\mu_\alpha \in [0,1],\ v_\alpha \in [0,1],\ \mu_\alpha + v_\alpha \leqslant 1,\ \pi_\alpha = 1 - \mu_\alpha - v_\alpha \tag{2}$$

and the complement of α is denoted by $\alpha^c = (v_\alpha, \mu_\alpha, \pi_\alpha)$.

For any two IFVs $\alpha_i = \left(\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i}\right)(i = 1,2)$, the following operational laws are valid [11,15]:

(1) $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1} \cdot \mu_{\alpha_2}, v_{\alpha_1} \cdot v_{\alpha_2}, (1 - \mu_{\alpha_1})(1 - \mu_{\alpha_2}) - v_{\alpha_1} \cdot v_{\alpha_2})$;

(2) $\lambda\alpha = (1 - (1 - \mu_{\alpha_1})^{\lambda}, v_{\alpha_1}^{\lambda}, (1 - \mu_{\alpha_1})^{\lambda} - v_{\alpha_1}^{\lambda})$.

Based on the operations (1) and (2), Xu [11] introduced an intuitionistic fuzzy weighted averaging (IFWA) operator as follows:

Definition 1. Let $\alpha_i = \left(\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i}\right)(i = 1,2,\ldots,n)$ be a collection of IFVs, an intuitionistic fuzzy weighted averaging (IFWA) operator of dimension n is a mapping IFWA: $\Omega^n \to \Omega$ that has an associated weighting vector $w = (w_1, w_2, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula: