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Age replacement policy for a two-unit system subject to non-homogeneous pure birth shocks

Shey-Huei Sheu [a,b,*], Chien-Kuo Sung [c], Tsung-Shin Hsu [b], Yan-Chun Chen [c]

[a] Department of Statistics and Informatics Science, Providence University, Taichung 43301, Taiwan
[b] Department of Industrial Management, National Taiwan University of Science and Technology, Taipei 10607, Taiwan
[c] Department of Industrial Engineering and Management, Tungnan University, New Taipei 22202, Taiwan

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ABSTRACT

When a two-unit system is subjected to shocks, it may fail due to interactions between the units. Two types of shocks stemming from a non-homogeneous pure birth process can affect a two-unit system. A Type I shock causes a minor failure of Unit A which affects Unit B and can be rectified with minimal repair. The Type II shock causes total (catastrophic) system failure, at which point an unplanned maintenance is required to replace the failed system. Two-unit systems also exhibit failure rate interactions between units: failure of Unit A causes an internal shock that increases the failure rate of Unit B, while failure of Unit B causes instantaneous failure of Unit A. The occurrence of a particular type of shock is based on a random mechanism dependent on the number of shocks that have occurred since the last replacement. The goal of this study is to derive the expected replacement cost rate by introducing relative costs as a factor, and determining the optimal replacement period T^* which minimizes cost. A numerical example is presented to illustrate application of the method.

1. Introduction

Effective preventive maintenance practices and unit replacement models can prevent unplanned failures and severe production losses. Replacement model design has been extensively studied, and such models have been applied widely to electronics, machinery and communications equipment, and industrial, environmental, military, and medical systems.

Many two-unit systems deteriorate with age, and random failures occur during operation that can be repaired. It is worthwhile to study optimal maintenance or replacement policies based on reliability theory in order to minimize operating costs and catastrophic breakdown risks. However, most previous studies of two-unit systems focus on systems subjected to shocks described by a non-homogeneous Poisson process (NHPP). The NHPP shock process, which is dependent only on the system age, is not applicable to systems that are affected by the number of failures. In this paper, we focus on shocks that occur according to a non-homogeneous pure birth process (NHPBP). The NHPBP is more appropriate for modeling a deterioration process that depends on both the system age and the number of shocks. Using the NHPBP shock process, we investigated maintenance or replacement policies for two-unit systems. This model can be applied to a wide variety of equipment to increase availability and reduce operating costs, including electrical networks, brakes, gearbox bearings, or airplane landing systems.

* Corresponding author. Address: Department of Statistics and Informatics Science, Providence University, No. 200, Chung Chi Rd., Taichung 43301, Taiwan. Tel.: +886 4 26328001x12232; fax: +886 4 26314846.

E-mail address: shsheu@pu.edu.tw (S.-H. Sheu).

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An Age Replacement Policy (ARP) is quite common and easy to implement. Under a classical ARP, a system in operation is replaced at age T or at total failure, whichever occurs first [1]. Maintenance policies, including both replacement and minimal repair, can be found in the literature. Nakagawa [2] and Chen [3] considered a system with two types of failures: Type I failures, which occur with probability q and are rectified by minimal repair, and Type II failures, which occur with probability p $(1-q)$ and require replacement. Sheu et al. [4] proposed an extended replacement policy with general random repair costs and age-dependent minimal repair [5,6]. Chang et al. [7] presented a replacement model with age-dependent failure type based on a cumulative repair-cost limit policy, whose concept used information on all repair costs to decide whether the system is repaired or replaced. Huynh et al. [8] examined the condition-based maintenance of single-unit systems which were subject to the competing and dependent failures due to deterioration and traumatic shock events.

When failures in multi-unit systems are statistically correlated, the units interact in different ways. Accordingly, if unit interaction in a multi-unit system is considered in designing a preventive maintenance policy, then interactions involving failure take three forms [9]. The first form is a Type I failure interaction, when a unit in a system fails causing other units to immediately fail at a particular probability. Murthy and Nguyen [10] first discussed failure interactions in two-unit systems and extended the results to multi-unit systems [11], while Jhang and Sheu [12] extended the fixed probability to a function of time t. Scarf and Deara [13] discussed an age-based opportunity replacement policy for addressing Type I failure interactions and economic dependence in a two-unit system, and further analyzed a block replacement policy for a two-unit system under the conditions of failure and economic dependence [14]. The second form is a failure rate interaction, where the failure of a unit acts as an internal shock that increases the failure rates of other units. This failure interaction is deemed Type II. Murthy and Nguyen [10] and Lai and Chen [15] examined a replacement policy for two-unit systems with interactions between failure rates [16]. The third form is shock damage interaction, when failure of a unit in a system randomly damages other units; the damage rapidly accumulates, causing system failure when the cumulative damage exceeds a threshold value. Nakagawa and Murthy [17] studied the optimal replacement policy in a two-unit system based on the number of unit 1 failures (N^*). Satow and Osaki [18] further developed the work of Nakagawa and Murthy by proposing a two-parameter (T, k) replacement policy for a two-unit system.

Studies of NHPP shocks and failure rate interactions in two-unit systems have considered only individual periodic [9] or discrete [19] replacement models, and a generalized replacement policy with two parameters (n, T) has been proposed by Sung et al.. This study presents an age replacement model that allows minimal repairs for two-unit systems which suffer NHPBP shocks and failure rates interact. Such a replacement model can be applied to many industrial systems, such as two-unit circuit systems where one receives a pulse shock and another suffers from an increased failure rate, as if it had received a shock as well. This model also can be illustrated by vehicular brake systems. An automobile or motorcycle decelerates due to friction between the brake pads and the rotor; brake pads and disc rotors are thus critical brake system components. If brake pads are worn out but not replaced, the pressure plates can cause serious damage to the rotors. With repetitive braking, disc rotors fail because of uneven and/or excessive wear, causing the entire brake system to fail. Excessive brake pad wear is considered the cause of the brake system failure, creating a failure interaction. Similarly, the units in a two-unit system may fail due to aging or shocks, and the failure rates of these units may interact. For example, Unit A may suffer a minor failure due to aging and Type I shocks, but Unit A failures can be rectified by performing minimal repairs. However, similar to an internal shock, Unit A failure increases the failure rate of Unit B, and Unit B may fail due to aging, failure rate interactions, or Type II shocks. When Unit B suffers a failure, the system fails entirely. This study examines an age replacement model for two-unit systems allowing minimal repairs that are affected by shocks due to NHPBPs and interacting failure rates. The T^* of the optimal replacement policy is found, and an illustrative numerical example is presented.

The key task for most replacement policies is to develop the expected cost function from which an optimization policy can be determined. Extensive studies exist on this topic. Chen and Savits [20] established the expected total α-discounted cost for a system under the ARP, including the assumption that shocks occur according to a compound non-homogeneous Poisson process [21]. Puri and Singh [22] considered the ARP optimization problem by taking into account non-monotone marginal cost functions.

Many works focus on systems subjected to shocks that cause system failures. Abdel-Hameed and Proschan [23] examined the case with shocks using an NHPP model. Boland and Proschan [24] investigated the optimal periodic replacement policy for a system subject to non-homogeneous Poisson shocks and a time-independent cost structure. Sheu [25] considered a more generalized age and block replacement problem under a non-homogeneous Poisson shock process, and determined optimal policy conditions. Chien et al. [26] proposed the ARP with minimal repair based on a cumulative repair cost limit and random lead time for replacement delivery. Finally, the life distribution of a system subject to a sequence of shocks occurring randomly in time according to a non-stationary pure birth process was considered by Abdel-Hameed and Proschan [27] and Sheu et al. [28,29].

In this paper, we consider a more general case in which shocks occur according to a non-homogeneous or non-stationary pure birth process. Specifically, we consider a maintenance policy that includes both replacement and repair. The decision to repair or replace a system at failure depends on both the type and the number of shocks suffered since the last replacement. The aim of our study is to examine the structure of the optimal maintenance policy.

The model is described at the beginning of Section 2, and the expected cost rate is obtained from this model. The optimal T^* that minimizes cost under certain conditions is discussed in Section 3. Section 4 presents a numerical example. Finally, Section 5 draws conclusions based on the previous sections.