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Elastodynamic analysis of a cracked orthotropic half-plane

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ABSTRACT

The solution of elastodynamic volterra-type dislocation in an orthotropic half-plane is obtained by means of the Fourier transforms. The distributed dislocation technique is used to construct integral equations for an orthotropic half-plane weakened by cracks where the domain is under time-harmonic anti plane traction. These equations are of Cauchy singular type at the location of dislocation which is solved numerically to obtain the dislocation density on the faces of the cracks. The dislocation densities are employed to determine stress intensity factors for multiple smooth cracks. Several examples are solved and the stress intensity factors for multiple cracks with different configuration are obtained.

1. Introduction

Composite materials are indispensable in many industries, necessitating the thorough comprehension of their behavior under load. The cracking of an orthotropic occurs during manufacturing or service life of a mechanical component and may be considered as the major cause of failure. Multiple cracks with any shape and direction may exist in the material making the analytical stress analysis of a body intractable. The stress analysis in elastic regions weakened by cracks subject to dynamic loading has drawn the attention of several researchers. Apparently, the first study dealing with dynamic crack problems was conducted by Maue [1]. He analyzed a semi-infinite crack in an infinite plane under time-harmonic stress wave by means of the Wiener–Hopf technique. The dynamic stress intensity factor for a finite crack in the infinite plane under anti-plane deformation was determined by Loeber and Sih [2]. The elastodynamic problem of the diffraction of stress waves by a crack near an interface was investigated by Luong et al. [3]. They analyzed the stress field near a crack within a half-plane which is bonded to a half-plane of a different material. Lin et al. [4], considered the cracked half-plane under time-harmonic excitation. The dynamic stress intensity factor of an edge crack was determined. Mendelsohn et al. [5], investigated the scattering of incident surface waves and incident body waves by a surface breaking crack in an elastic half-plane. The plane strain problem for an elastic half-plane containing a rigid flat inclusion and subjected to a concentrated harmonic surface load was determined by Doyum et al. [6]. The diffraction problem by two collinear cracks located in an orthotropic medium subjected to time-harmonic stress waves was investigated by Itou [7]. Itou et al. [8], considered the diffraction of incident harmonic stress waves by two parallel cracks in an infinite orthotropic medium. Dos et al. [9] considered the diffraction of shear waves by Griffith crack in an infinite transversely orthotropic medium. Meguid and Wang [10], investigated the failure behavior of fiber reinforced composites involving cracked matrix and imperfectly bounded fibers under dynamic anti-plane excitation. Results show the effect of the frequency of the incident wave upon the dynamic stress intensity factors of the cracked matrix and interaction between a main crack and a completely

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Nomenclature

a	half lengths of straight crack
a, b	lengths of major and minor semi-axes of elliptical crack
A_{ij}, C_{ij}, D_{ij}	coefficients matrix
$A_1,(\lambda),B_1(\lambda),A_2(\lambda)$	unknowns coefficients
b_z	Burgers vector
B_{zj}	complex dislocation densities
B_{1j}, B_{2j}	real and imaginary parts of dislocation densities
c	hear wave velocity
$Ci(x)$, $Si(x)$	sine and cosine integral functions
$g_{ij}\,(p)$	regular terms of dislocation densities
$H(x)$	Heaviside step function
$J_p\,(x)$, $Y_p\,(x)$	Bessel functions of first and second kinds of order p
k_{Li}, k_{Ri}	stress intensity factors of left and right side of crack
k_0	stress intensity factor of a crack in infinite plane
$k_{ij}\,(s, p)$	kernel of integral equations
k_{ij1}, k_{ij2}	real and imaginary parts of kernel of integral equations
k_T	wave number
N	total number of cracks
r_{Li}, r_{Ri}	distance from right and left crack tips
w	out of plane displacement component
w	amplitude of displacement component
x, y	coordinates
$\alpha_i(s)$, $\beta_i(s)$	functions describing the geometry of cracks
$\delta(\lambda)$	Dirac delta function
δ_{ij}	Kronecker delta
$\theta_i\,(s)$	crack orientation
G_{zx}, G_{zy}	orthotropic shear moduli of elasticity of material
ρ	mass density
σ_{1nz}, σ_{2nz}	real and imaginary parts of traction vector
σ_{zx}, σ_{zy}	out of plane stress components
ω	angular frequency

debounded fiber which can be modeled as a cavity. Sun et al. [11], investigated the treatment of the dynamic behavior of interacting cracks in a piezoelectric layer bonded with two dissimilar piezoelectric half-planes under harmonic anti plane shear waves. By utilizing the Fourier transformation the problem reduced to two pairs of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surfaces. Results show the effect of the geometry of the cracks, the frequency of the incident waves and materials properties upon the dynamic stress intensity factors. The solution procedures devised in all above studies are neither capable of handling curved cracks among multiple cracks with arbitrary arrangement. Moharrami and Ayatollahi [12], investigated the scattering of anti-plane harmonic stress waves by multiple cracks in an orthotropic plane.

The primary objective of this study is to apply the distributed dislocation technique for the stress analysis of multiple cracks with arbitrary patterns in an orthotropic half-plane under time-harmonic point load. The solution procedures devised in all above studies are neither capable of handling curved cracks among multiple cracks with arbitrary arrangement under dynamic loading. The complex Fourier transform is employed to obtain transformed displacement and stress fields. The inversion of transformed displacement and stress fields is carried out by changing the contour of integration. The dislocation solutions are then used to formulate integral equations for a half-plane weakened by several cracks. The integral equations are of Cauchy singular types which are solved numerically for the dislocation density on the cracks faces. Several examples of cracks are solved to study the effects of excitation frequency on the stress intensity factor of cracks to illustrate the applicability of the procedure.

2. Formulation of the problem

The distributed dislocation technique is an efficient means for treating multiple curved cracks with smooth geometry. The major obstacle in the utilization of the method is the knowledge of stress fields due to a single dislocation in the region. We now take up this task for an orthotropic half-plane containing a screw dislocation under time-harmonic excitation. For a medium under anti-plane deformation, the only nonzero displacement component is the out of plane component $W(x, y, t)$. Consequently, the constitutive relationships are: