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Optimizing the rank position of the DMU as secondary goal in DEA cross-evaluation

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ABSTRACT

Cross-efficiency evaluation is an extension of Data Envelopment Analysis (DEA) that permits not only the determination of a ranking of Decision Making Units (DMUs) but also the elimination of unrealistic weighting schemes, thereby rescinding the necessity for the inclusion of individual judgements in the models. The main deficiency of the procedure is the non-uniqueness of the optimal weights, which results in the peer evaluations dependences, for instance, on the software used to determine DMU's efficiencies. This shortfall justifies the inclusion of secondary goals in order to determine cross-efficiency values. In this paper a new proposal of a secondary goal is studied. The idea is related with that proposed in Wu et al. (2009), in which the objective is the optimization of the rank position of the DMU under evaluation. In the procedure proposed here, an incentive to break level-pegging ties between alternatives is introduced by considering that efficiency scores induce a weak order of alternatives. The model is illustrated with a preference-aggregation application.

1. Introduction

Data Envelopment Analysis (DEA) is a nonparametric technique originally developed to evaluate the efficiency of non-profit-making organizations (Decision Making Units, DMUs, in DEA nomenclature), which produces multiple outputs by using multiple inputs. In essence, DEA models are used in an effort to determine the relative efficiency of each DMU with respect to the production function (whose specific form may remain undetermined) by means of solving linear problems.

However, DEA has widely exceeded the original idea it was conceived for in the initial paper of Charnes et al. [1]. Nowadays, DEA can be seen as a technique for the analysis of a wide range of multicriteria problems. See, for example [2] for a complete revision of DEA model applications.

The original models classify DMUs into two sets on the basis of efficiency scores: efficient units (those that yield a DEA score equal to unity) and non-efficient units (whose efficiency score is lower or greater than one, depending on the orientation of the model). In this context, increasing interest in DEA literature follows the development of procedures which increase this discrimination since, in many cases, a natural requirements is the determination of which unit is the best from among those classified as efficient, or the determination of a rank order of the complete set of DMUs. Initially, there is no criterion to affirm that one efficient DMU is better than any other (all of them yield the same efficiency score equal to unity). Therefore, in these situations, an additional procedure or modification over the DEA model is required in order to discriminate between efficient DMUs.

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There are many examples of procedures which attempt to increase the discrimination between DMUs. A complete revision of these procedures can be found in [3]. In this family of procedures can be cited, among others, the Superefficiency methodology proposed in [4], the inclusion of additional information, that allow to include human judgements into DEA models or simply reduce the allowable solution space to avoid unrealistic solutions (see, among others the proposal of [5]), and the cross-efficiency evaluation. The focus of this paper is on the improvement of the cross-efficiency evaluation.

The main idea in cross-evaluation is to use DEA in a peer evaluation instead of a self-evaluation. Each DMU is evaluated not only with respect to the weights obtained in self-evaluation, the set of weights allows the unit under evaluation to obtain the best efficiency score, but also with respect to the optimal sets of weights of the other DMUs. A matrix of cross-evaluation scores is obtained and the units are evaluated with respect to a cross-efficiency value, obtained as the average of the scores with respect to the optimal set of weights of every DMU.

The main advantages of cross-efficiency evaluation are twofold. Firstly, it provides an order for DMUs, including a rank order of those units which are efficient in the self evaluation. Secondly, the procedure eliminates unrealistic weighting schemes without requiring the elicitation of weight restrictions from experts [6]. This means that the cross-efficiency evaluation can be found in a significant number of applications in DEA literature. A review of these applications can be seen, among others, in [7,8].

Two main aspects have focused the attention in this field: the selection of an appropriate aggregation function (in order to obtain a unique value from the multiple cross evaluations) and the consideration of a secondary goal to discriminate among multiple solutions. This latter aspect will be studied in this work.

The main drawback to the cross-efficiency evaluation is the non-uniqueness of optimal weights obtained from DEA models. That is to say, there can be many sets of weights that permit a DMU to yield its best efficiency score. However, the evaluation of the other units will depend on which vector is finally selected. The inclusion of a secondary goal supposes, implicitly, a procedure to discriminate among multiple optimal weights, selecting the best option from all the weights that permit the DMU under consideration to achieve its maximum value.

Nevertheless, in this family of models the aggregated values finally attained to each DMU prevail over the selection of a particular vector of weights. The objective of the models used to be the discrimination between efficient DMUs or the construction of a ranking, goals in which the most important aspect in the value attained to each alternative and not the relative importance of each variable in this aggregated value.

Since the initial development of the idea of cross-efficiency evaluation in [9,10], secondary goals have been proposed in order to select a singular vector of weights. The authors differentiate between aggressive and benevolent formulation. The idea inherent in these formulations is to choose not only the weights that maximize the efficiency of the DMU under evaluation but also to simultaneously maximize or minimize the average efficiency of the other units for the cases of benevolent and aggressive formulation, respectively.

Several papers have carried out research in this field, improving the original proposals or including new ideas for the secondary goals. In [11] the authors proposes new formulation for the benevolent and aggressive goals, with easier ways to implement the initial models. In [12] a profiling method is proposed. The authors extend the idea of [13] to evaluate each DMU according to the optimal weighting scheme of other DMUs, but take only one input into account at a time and only with respect to its related outputs. The result is a profile for each DMU. A secondary goal that refines the aggressive formulation is proposed in [8] which prioritizes the rank position of the DMU under evaluation over above the efficiency score. In [7], a neutral model is studied that focuses the secondary goal on the DMU under evaluation, without maximizing or minimizing the efficiency of the remaining units. In [14] a symmetric weight assignment technique is proposed, in order to make an adequate selection of weights in the cross-efficiency evaluation.

In [15] new secondary criteria are proposed. The authors defined the concept of *d-inefficiency* as the deviation variables of DMU d in the original CCR model ([1]) in such a way that the dth DMU is efficient if the deviation is zero and 1 minus the deviation can be regarded as a measure of inefficiency. In this context the authors establish three criteria for the cross-evaluation: minimize the total deviation, minimize the maximum deviation across DMUs, and minimize the mean absolute deviation of the DMUs.

The main goals proposed in some previous papers are reached via the minimization (or maximization) of the efficiency of the rest of the DMUs taken as a whole. One of the criteria proposed in [11] allows the DMU to focus its aggression (or benevolence), DMU by DMU. The authors defined this evaluation as targeted aggression (benevolence). In this paper, a new proposal along these lines is studied. A new idea for the determination of cross-evaluation is proposed in which the attention is focused on the determination of the set of weights that not only encounters the best CCR-efficiency score of the DMU under evaluation but also minimizes the efficiency score of the closest DMU in terms of efficiency.

The rest of the paper is organized as follows. Section 2 is devoted to the presentation of the problem and the description of cross-efficiency evaluation in the DEA context. Section 3 includes the new proposal for the secondary goal in cross-efficiency evaluation. In Section 4, an illustrative example is presented while Section 5 is devoted to conclusions.

2. The cross-efficiency evaluation

Consider a set of n DMUs, each producing s different outputs from m different inputs. The expressions x_{ij} ($i = 1,\ldots,m$) and y_{rj} ($r = 1,\ldots,s$) denote the ith input utilized and rth output produced by DMU j, respectively, with $j = 1,\ldots,n$.