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Anti-plane elastodynamic analysis of orthotropic planes weakened by several cracks

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ABSTRACT

Stress analysis is carried out in an orthotropic plane containing a Volterra-type dislocation, the distributed dislocation technique is employed to obtain integral equations for an orthotropic plane weakened by cracks under time-harmonic anti-plane traction. The integral equations are of Cauchy singular type at the location of dislocation which are solved numerically. Several examples are solved and the stress intensity factors for multiple cracks with different configuration are obtained.

1. Introduction

In composite materials, defects in the form of cracks generate regions of high stress gradient. The cracking of orthotropic materials occurs during manufacturing or service life of a mechanical component and may be considered as the major cause of failure. Multiple cracks with any shape and direction may exist in the material making the analytical stress analysis of a body intractable. The stress analysis in elastic regions weakened by cracks subject to dynamic loading has drawn the attention of several researchers. Apparently, the first study dealing with dynamic crack problems was conducted by Maue [1]. He analyzed a semi-infinite crack in an infinite plane under time-harmonic stress wave by means of the Wiener–Hopf technique. The dynamic stress intensity factor for a finite crack in the infinite plane under anti-plane deformation was determined by Loeber and Sih [2]. The diffraction problem by two collinear cracks located in an orthotropic medium subjected to time-harmonic stress waves was investigated by Itou [3]. Itou and Haliding [4], considered the diffraction of incident harmonic stress waves by two parallel cracks in an infinite orthotropic medium. Dos et al. [5] considered the diffraction of shear waves by Griffith crack in an infinite transversely orthotropic medium. Meguid and Wang [6], investigated the failure behavior of fiber reinforced composites involving cracked matrix and imperfectly bounded fibers under dynamic anti-plane excitation. Results show the effect of the frequency of the incident wave upon the dynamic stress intensity factors of the cracked matrix and interaction between a main crack and a completely debounded fiber which can be modeled as a cavity. Ma et al. [7], investigated the scattering of anti-plane harmonic waves by a finite crack in the functionally graded orthotropic medium. By utilizing the Fourier transformation the problem reduced to a pair of dual integral equations which were solved by series expansion method. Results show the effect of material properties upon the dynamic fracture behavior of functionally graded materials. The solution procedures devised in all above studies are neither capable of handling curved cracks among multiple cracks with arbitrary arrangement. Ayatollahi et al. [8], investigated the scattering of anti-plane harmonic stress waves by multiple defects in an infinite isotropic plane.

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Nomenclature

a	half lengths of straight crack
a, b	lengths of major and minor semi-axes of elliptical crack
A_{ij}, C_{ij}, D_{ij}	coefficients matrix
$A(\xi), B(\xi)$	unknowns coefficients
b_z	Burgers vector
B_{zj}	complex dislocation densities
B_{1j}, B_{2j}	real and imaginary parts of dislocation densities
c	shear wave velocity
$Ci(x), Si(x)$	sine and cosine integral functions
$g_{ij}(p)$	regular terms of dislocation densities
$H(x)$	Heaviside step function
$H_p^2(x)$	Hankel function of the second kind of order p
$J_p(x), Y_p(x)$	Bessel functions of first and second kinds of order p
k_{Li}, k_{Ri}	stress intensity factors of left and right side of crack
k_0	stress intensity factor of a crack in infinite plane
$k_{ij}(s, p)$	kernel of integral equations
k_{ij1}, k_{ij2}	real and imaginary parts of kernel of integral equations
k_T	wave number
N	total number of cracks
r_{Li}, r_{Ri}	distance from right and left crack tips
W	out of plane displacement component
w	amplitude of displacement component
w_0	amplitude of displacement disturbance
x, y	coordinates
$\alpha_i(s), \beta_i(s)$	functions describing the geometry of cracks
γ	constant in displacement disturbance
$\delta(\xi)$	Dirac delta function
δ_{ij}	Kronecker delta
$\theta_i(s)$	crack orientation
G_{zx}, G_{zy}	orthotropic shear moduli of elasticity of material
ρ	mass density
$\sigma_{1nz}, \sigma_{2nz}$	real and imaginary parts of traction vector
σ_{zx}, σ_{zy}	out of plane stress components
ϕ	angle of disturbance with the y-axis
ω	angular frequency

The primary objective of this study is to apply the distributed dislocation technique for the stress analysis of multiple cracks with arbitrary patterns in an orthotropic plane under time-harmonic anti-plane traction. The complex Fourier transform is employed to obtain transformed displacement and stress fields. The inversion of transformed displacement and stress fields is carried out by changing the contour of integration. The dislocation solutions are then used to formulate integral equations for a plane weakened by several cracks. The integral equations are of Cauchy singular types which are solved numerically for the dislocation density on the cracks faces. The only limitation of the procedure is that the crack closing is not considered. Therefore, the applied traction on the cracks faces should not allow the occurrence of even partial closing of cracks. To confirm the validity of formulations, numerical values of dynamic stress intensity factors for a crack is compared with the results in literature. Several examples of cracks are solved to study the effects of excitation frequency and angle of incidence on the stress intensity factor of cracks to illustrate the applicability of the procedure.

2. Orthotropic plane with screw dislocation

The distributed dislocation technique is an efficient means for treating multiple curved cracks with smooth geometry. However, determining stress fields due to a single dislocation in the region has been a major obstacle to the utilization of this method. We now take up this task for an orthotropic plane containing a screw dislocation under time-harmonic excitation. For a medium under anti-plane deformation, the only nonzero displacement component is the out of plane component $W(x, y, t)$. Consequently, the constitutive relationships are

$$\begin{aligned} \sigma_{zx}(x, y, t) &= G_{zx}\frac{\partial W}{\partial x}, \\ \sigma_{zy}(x, y, t) &= G_{zy}\frac{\partial W}{\partial y}. \end{aligned} \tag{1}$$