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Free vibration characteristics of a functionally graded beam by finite element method

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ABSTRACT

This paper presents the dynamic characteristics of functionally graded beam with material graduation in axially or transversally through the thickness based on the power law. The present model is more effective for replacing the non-uniform geometrical beam with axially or transversally uniform geometrical graded beam. The system of equations of motion is derived by using the principle of virtual work under the assumptions of the Euler–Bernoulli beam theory. The finite element method is employed to discretize the model and obtain a numerical approximation of the motion equation. The model has been verified with the previously published works and found a good agreement with them. Numerical results are presented in both tabular and graphical forms to figure out the effects of different material distribution, slenderness ratios, and boundary conditions on the dynamic characteristics of the beam. The above mention effects play very important role on the dynamic behavior of the beam.

1. Introduction

Recently, a new class of composite materials known as functionally graded materials (FGMs) has drawn considerable attention. This material is a type of material whose composition designed to change continuously within the solid. The gradient compositional variation of the constituents from one surface to the other provides an elegant solution to the problem of high transverse shear stresses that are induced when two dissimilar materials with large difference in material properties are bonded. The FGM concept originated in Japan in 1984 during the space-plane project, in the form of a proposed thermal barrier material capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 K across a cross-section <10 mm [1]. Since FG beams are used in aerospace, automotive industries and machine elements, understanding their dynamic behavior is important. Compared with FG plates and shells, studies for FG beams are relatively less [2].

Cheng and Batra [3] exploited Reddy's third-order plate theory to study buckling and steady state vibrations of a simply supported functionally gradient isotropic polygonal plate resting on a Winkler Pasternak elastic foundation and subjected to uniform in-plane hydrostatic loads. Sankar [4] presented an elasticity solution for simply supported FG beams subjected to sinusoidal transverse loading. The Young's modulus is assumed to vary in an exponential fashion through the thickness. Snakar found when the softer side is loaded; the stress concentrations are less than that in a homogeneous beam, and the reverse is true when the harder side is loaded. Chakraborty et al. [5] developed a new beam element to study the thermoelastic behavior of functionally graded beam structures. The element is based on the first-order shear deformation theory and it accounts for varying elastic and thermal properties along its thickness. Both exponential and power-law variations of material property distribution are used to examine different stress variations. Static, free vibration and wave propagation

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problems are considered. Goupee and Senthil [6] proposed a genetic algorithm methodology to optimize the natural frequencies of functionally graded structures by tailoring their material distribution for three model problems. In the first problem, the material distributions that maximize each of the first three natural frequencies of a functionally graded beam were required. The goal of the second model problem was to minimize the mass of a functionally graded beam while constraining its natural frequencies to lie outside certain prescribed frequency bands. The last problem aimed to minimize the mass of a functionally graded beam by simultaneously optimizing its thickness and material distribution such that the fundamental frequency is greater than a prescribed value. Aydogdu and Taskin [2] investigated analytically free vibration of simply supported FG beam. Young's modulus of beam varies in the thickness direction according to power law and exponential law. It is found that classical beam theory (CBT) gives higher results and difference between CBT and higher order theories is increasing with increasing mode number.

Semi-analytical elasticity solutions for static bending and thermal deformation of bi-directional FG beams are presented using a hybrid state space-based differential quadrature method by Lu et al. [7]. The results illustrated the ability of the bi-directional functionally graded properties along the axial direction to reduce thermal stresses instead of the conventional unidirectional functionally graded materials. Li [8] presented a new unified approach for analyzing the static and dynamic behaviors of functionally graded beams (FGB) with the rotary inertia and shear deformation. The Timoshenko beam theory is extended to treat FGB as well as layered beams. Different from previous approaches, a single fourth-order partial differential equation has been derived. Xiang and Yang [9] investigated free and forced vibration of a thermally prestressed, laminated functionally graded beam of variable thickness using the Timoshenko beam theory and the differential quadrature method. It is shown that the use of thicker FGM layers with a smaller volume fraction index in the laminated beam structure effectively increases natural frequencies and lowers the vibration amplitude. Ying et al. [10] presented two-dimensional elasticity solutions for bending and free vibration of functionally graded beams resting on Winkler–Pasternak elastic foundations. Trigonometric series are adopted for the fully simply-supported beams to transform the partial differential state equation into an ordinary one, thus making exact solutions possible. A third-order zigzag theory based model for layered functionally graded beams in conjunction with the modified rule of mixtures (MROM) for effective modulus of elasticity is validated through experiments for static and free vibration response by Kapuria et al. [11]. This study demonstrated the capability of the zigzag theory in accurately modelling the mechanics of layered beams with the ceramic content.

Piovan and Sampoia [12] studied the dynamic behavior of rotating beams made of functionally graded materials. The model has been deduced employing a formulation accounting for shear-deformability and nonlinear strain–displacements relationships. The influence of the graded properties in the damping effects and geometric stiffening of the rotating beam by using the finite element method are figured out. Sina et al. [13] developed a new beam theory used to analyze free vibration of functionally graded beams. The beam properties are assumed to be varied through the thickness following a simple power law distribution. The results showed that the new theory is a little different in natural frequency from the traditional first-order shear deformation beam theory and the mode shapes of the two methods are coincidental. Oyekoya et al. [14] exploited Mindlin-type element and Reissner-type element for the modelling of FG composite plate subjected to buckling and free vibration. Vibration and buckling analysis were then undertaken for different fiber distribution cases and the effects of fiber distribution were studied. Simsek and Kocaturk [15] analyzed free vibration characteristics and the dynamic behavior of a FG simply-supported beam under a concentrated moving harmonic load. The system of equations of motion is derived by using Lagrange's equations under the assumptions of the Euler–Bernoulli beam theory. It is observed that, the effects of the different material distribution, velocity of the moving harmonic load, the excitation frequency on the dynamic responses of the FG beam play very important role on the dynamic behavior of the FG beam.

Simsek [16] investigated dynamic behavior of a functionally graded beam under a moving mass is within the framework of Euler–Bernoulli, Timoshenko and the third-order shear deformation beam theories. Malekzadeh et al. [17] developed a formulation for the out-of-plane free vibration analysis of functionally graded (FG) circular curved beams in thermal environment. The formulation is based on the first-order shear deformation theory (FSDT). The material properties are assumed to be temperature dependent and graded in the direction normal to the plane of the beam curvature. It was shown that the temperature dependence of the material properties has significant effects on the natural frequency parameters. Ke et al. [18] investigates the nonlinear free vibration of functionally graded nanocomposite beams reinforced by single-walled carbon nanotubes (SWCNTs) based on Timoshenko beam theory and von Karman geometric nonlinearity. The material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) are assumed to be graded in the thickness direction and estimated though the rule of mixture. Results showed that; an increase in CNT volume fraction leads to higher linear and nonlinear frequencies for both uniform distribution and FG-CNTRC beams. Both linear and nonlinear frequencies of FG-CNTRC beam with symmetrical distribution of CNTs are higher than those of beams with uniform or unsymmetrical distribution of CNTs. Huang and Li [19] studied free vibration of axially functionally graded beams with non-uniform cross-section. They transformed the governing equation with varying coefficients to Fredholm integral equations to find the natural frequencies of beams with variable flexural rigidity and mass density.

In the present study, the free vibration analysis of FG beams is investigated using numerical finite element method. The equations of motion of FG beams are derived using Euler–Bernoulli beam theory and virtual work principle. The material constituents of beams assumed to be varying through the thickness or longitudinal directions according to a simple power law. The present model is effective for comparing the tapered beam of linearly variable width or depth and graded beams of special polynomial non-homogeneity. Finally, the effects of various boundary conditions (BC's), power-exponent index and beam's slenderness ratio are investigated.