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Reduced order fully coupled structural–acoustic analysis via implicit moment matching

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ABSTRACT

A reduced order model is developed for low frequency, undamped, fully coupled structural–acoustic analysis of interior cavities backed by flexible structural systems. The reduced order model is obtained by applying a projection of the coupled system matrices, from a higher dimensional to a lower dimensional subspace, whilst preserving essential properties of the coupled system. The basis vectors for projection are computed efficiently using the Arnoldi algorithm, which generates an orthogonal basis for the Krylov Subspace containing moments of the original system. The key idea of constructing a reduced order model via Krylov Subspaces is to remove the uncontrollable, unobservable and weakly controllable, observable parts without affecting the transfer function of the coupled system. Three computational test cases are analyzed, and the computational gains and the accuracy compared with the direct inversion method in ANSYS.

It is shown that the reduced order model decreases the simulation time by at least one order of magnitude, while maintaining the desired accuracy of the state variables under investigation. The method could prove as a valuable tool to analyze complex coupled structural–acoustic systems, and their subsequent optimization or sensitivity analysis, where, in addition to fast analysis, a fine frequency resolution is often required.

1. Introduction

In a modern passenger vehicle or a commercial aircraft the noise, vibration and harshness (NVH) performance is one of the key parameters which the customer uses to assess product quality. In order to gain competitive advantage, manufacturers are continually striving to reduce NVH levels. As a result, design engineers often seek to evaluate the low frequency NVH behavior of automotive/aircraft interiors using coupled finite element-finite element (FE/FE) or finite element-boundary element (FE/BE) discretized models. Due to the coupling between the fluid and structural domains in the coupled displacement/pressure (u/p) FE/FE formulation [1–3], the resulting mass and stiffness matrices are no longer symmetrical. In addition to this, at least 6–8 linear elements per wavelength are required to give reasonable prediction accuracy for structural–acoustic problems [4–7]. This can result in huge model sizes for higher frequency analysis, and hence a significant increase of computational time and expense. Furthermore, this presents as a major problem where optimization is required, especially when there are a large number of design variables to be optimized. Therefore, generation of compact models, for fast coupled structural–acoustic analysis is of great interest to the NVH community.

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Alternative FE/FE formulations leading to positive-definite, symmetric matrices do exist, and can be classified by unknown fluid variables taking the form of velocity potential [8,9], a combination of velocity potential and sound pressure level [10], displacements in the fluid [11] and a combination of displacements and pressures [12]. However, these formulations encounter new problems such as spurious rotational modes, increase in dimension of the problem size or involves solving a complex dynamic matrix for an undamped coupled structural–acoustic problem. Such problems with these other formulations have resulted in the *Eulerian* (u/p) formulation [1–3] being adopted as the most appropriate prediction technique [7], and being widely employed in commercially available FE codes such as ANSYS, MSC/Nastran, DSS ABAQUS, LMS SYSNOISE.

The two most popular approaches to reduce the computational time of such coupled problems are mode superposition and the component mode synthesis (CMS) methods. The former method uses the dominant natural frequencies and mode shapes, extracted from a normal modal analysis and the response is assumed to be a linear combination of these modes. However, the standard mode superposition method, suffers from four major drawbacks: (a) The computation of coupled modes using a non-symmetric eigen-solver tends to be computationally very demanding [13]. (b) The second drawback is the treatment of damping. For well damped structures, a spatially distributed damping – often varying with frequency, has to be utilised [14]. (c) If a structural–acoustic optimization problem in a particular frequency band is considered, there exists the problem of only the higher order modes being truncated, leading to unwanted estimation of lower order modes repeatedly [15] and (d) The number of modes required to represent the frequency band under investigation is often an approximate guess of between $1.2\omega_E$ to $2\omega_E$ where ω_E is the upper frequency range [16]. In the modal synthesis or the CMS type approach, the system is divided into different components (structure and fluid), and the uncoupled normal modes from a symmetric eigenvalue problem are calculated. This set can then be treated as vectors for projection in the standard modal superposition model. However, the efficiency of the CMS type formulation for coupled structural–acoustic problems in reducing model size is poor, since a large number of acoustic modes are required to enforce displacement continuity along the fluid–structure interface. This is mainly due to the fact that the modes of the uncoupled acoustic model have a zero normal fluid displacement along the fluid–structure coupling interface. Therefore, for an accurate representation of the near-field effects in the vicinity of the fluid–structure coupling interface a large number of high-order modes in the acoustic modal basis is required, resulting in slow convergence of the coupled problem. This has been numerically demonstrated for a 1D tube example [6]. Further, the effect of the kept (retained) *dry* modes is critical to the convergence in the uncoupled modal basis method, especially for *strongly coupled* problems (e.g. presence of heavy fluid) [17–19].

The restriction of these two methods has left NVH engineers with a very limited number of tools for the numerical analysis of vehicle/aerospace interior noise prediction problems, and hence they are often forced to resort to mixed experimental–numerical approaches [20]. Other approaches to reduce computational time for structural–acoustic problems include the use of symmetrization techniques [21,17,19,22], geometric mesh skinning, the variation to the CMS approach namely the Automated Multi-level Substructuring [23,22], generation of Ritz vectors [21], use of acoustic influence co-efficients from BE models [24], truncated FE/FE analysis [15], the patented Acoustic Transfer Vector (ATV) method [25] and the use of *look-ahead* Lanczos process for symmetric matrices [26,27], to name a few. The reader is referred to the reviews by [4,17,14,28–31] for a description of some of the above mentioned approaches.

More recently, model order reduction (MOR), via implicit moment matching, has received considerable attention among mathematicians and the circuit simulation community [32,33]. It has been shown in various engineering applications [34–38], that the time required to solve reduced order models via implicit moment matching is significantly reduced when compared to solving the original higher dimensional problem and whilst maintaining the desired accuracy of the solution. The aim of MOR is to construct a reduced order model, from the original higher dimensional problem, which gives a faithful representation of the system input/output behavior (i.e. the transfer function) at particular points in the frequency domain. The reduction is achieved by applying a projection from a higher order to a lower order space using a set of Krylov Subspaces, generated by the Arnoldi algorithm. Additionally, the reduced model preserves certain essential properties such as maintaining the second order form and stability. Therefore, it would be possible to carry out investigations on the original system by replacing it with a reduced order model and undertake design modifications for a much smaller computational cost and time.

The paper focuses on the application of such Krylov-based MOR techniques to undamped, fully coupled structural–acoustic problems. The remainder of the paper is laid out as follows: In Section 2 the general framework for model order reduction for second order systems and its application to the coupled structural–acoustic problem is described. The Arnoldi algorithm adapted for model order reduction for the fully coupled structural–acoustic problem is presented. In Section 3 three numerical examples are solved by the direct approach using ANSYS finite element (FE) code [39], and by MOR via the moment matching Arnoldi approach. Simple models for error estimates and thus the convergence properties of MOR via the Arnoldi process are discussed. Further, the accuracy and computational times from the numerical test cases considered in this paper are discussed. Finally, Section 4 concludes the paper with a short discussion of the potential applications of MOR in structural acoustics, and future recommendations.

2. Model order reduction for second order systems

After discretization in space of a general mechanical system model, one obtains a system of ordinary differential equations of second order in matrix form as follows [40]: