Applied Mathematical Modelling 37 (2013) 916–938

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A unified formulation of PVD-based finite cylindrical layer methods for functionally graded material sandwich cylinders

Chih-Ping Wu *, Cheng-Hsin Kuo

Department of Civil Engineering, National Cheng Kung University, Tainan 70101, Taiwan, ROC

A R T I C L E   I N F O

Article history:
Received 4 October 2011
Received in revised form 29 February 2012
Accepted 12 March 2012
Available online 21 March 2012

Keywords:
Finite layer methods
The principle of virtual displacements
Bending
Vibration
Functionally graded materials
Cylinders

A B S T R A C T

A unified formulation of finite cylindrical layer methods (FCLMs) based on the principle of virtual displacements (PVDs) is developed for the quasi-three-dimensional (3D) bending and free vibration analyses of simply-supported, functionally graded material (FGM) sandwich circular hollow cylinders, in which the material properties of the FGM layer are assumed to obey the power-law distributions of the volume fractions of the constituents through the thickness coordinate. In this formulation, the cylinder is divided into a number of cylindrical finite layers, where the trigonometric functions and Lagrange polynomials are used to interpolate the in- and out-of-surface variations of the displacement components of each individual layer, respectively. Because an *h*-refinement is adopted in this article to yield the convergent solutions, the relative orders used for expansion of the displacement components remain variable, and can be freely chosen as linear, quadratic and cubic ones. The accuracy and convergence rate of a variety of PVD-based FCLMs developed in this article are assessed by comparing their solutions with the available 3D ones.

1. Introduction

Sandwich composite structures, consisting of a thick and soft homogeneous core bounded with two thin and stiff inner and outer face sheets, have being widely used in various engineering applications, such as aircraft, spacecraft, ship and automotive vehicle structures. The development of the theoretical methodologies and computational modeling of these structures has therefore received considerable attention [1–9], and their design, analysis, and construction have been discussed in several well-known books [10–12].

Some drawbacks have also been reported in the use of multilayered and sandwich composite structures, such as face sheet/core de-bonding and interface cracking, due to the fact that both the transverse stress concentration and in-surface stress discontinuity always occur at the interfaces between adjacent layers resulting from the mismatch of material properties. To enhance the resistance of conventional sandwich structures to the above-mentioned failures, a new class of functionally graded materials (FGMs) has been introduced as the constituents of the core-layer of these structures, the material properties of which gradually and continuously vary through the thickness coordinate.

Kashtalyan and Menshykova [13] and Woodward and Kashtalyan [14] presented a three-dimensional (3D) elasticity analysis of sandwich panels with an FGM core using the Plevako approach, in which the material properties of the core were assumed to obey an exponent-law exponentially varying through the thickness coordinate. The Plevako approach was also used in the 3D bending analysis of single-layered FGM plates and FGM coating/substrate plates by Kashtalyan [15] and

* Corresponding author. Fax: +886 6 2370804.
E-mail address: cpwu@mail.ncku.edu.tw (C.-P. Wu).

0307-904X/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2012.03.025

Kashtalyan and Menshykova [16], respectively. Pan [17] presented the 3D analysis of multilayered composite and functionally graded (FG) elastic material plates using the Stroh formalism [18]. Other articles dealing with 3D mechanics problems of sandwich composite and FGM plates and shells were carried out using the modified Pagano [19,20], state space [21–24], series expansion [25–28] and perturbation methods [29–32], the solutions of which can provide a standard for assessing the ones obtained using the approximate 3D and 2D theories.

The static, thermo-bending, vibration and buckling problems of another type of FGM sandwich plate, consisting of a homogeneous core-layer bounded with two FGM face-sheet layers, were studied by Zenkour [33–36] using a variety of equivalent single-layer theories (ESLTs), in which the material properties of the face-sheet layers were assumed to obey a power-law distribution of the volume fractions of the constituents through the thickness coordinate. The above-mentioned mechanical problems of multilayered FGM structures were also investigated by Ramirez et al. [37] using a discrete layer approach, Carrera et al. [38] using refined and advanced models, and Wu and Li [39] using a mixed third-order shear deformation theory, as well as Wu et al. [40], Wu and Chiu [41] and Wu and Yang [42] using the meshless collocation and element-free Galerkin methods.

Carrera [43] developed a compact notation for the analysis of composite structures. In a subsequent article [44] this approach was used to analyze generic loading conditions. In Ref. [45] this methodology was named "Carrera's Unified Formulation" (CUF) and a further generalization, called "Generalized Unified Formulation" (GUF) was presented. In Refs. [46,47] Carrera and coauthors investigated various refined and mixed theories for multilayered composite and FGM structures. Carrera et al. [48] and Cinefra et al. [49] developed a variable kinematic model and introduced it into CUF and GUF for the analysis of FGM plates and shells. Demasi [50–54] extended GUF to investigate PVD- and RMVT-based theories, where various global and mixed first-order and higher-order shear deformation, zig-zag, layerwise theories were included by defining the relative orders used for expansion of the primary variables. A comprehensive comparison among the 2D, quasi-3D and exact 3D solutions of sandwich plates was carried out by Demasi [55]. Finally, Carrera and Petrolo [56] adopted the penalty method to remove some terms of the CUF expansions and analyze the case in which different orders of expansion could be adopted. They also provided some guidelines and recommendations for the development of theories for metallic and composite plates.

Based on the PVD, Cheung and Jiang [57] developed a finite layer method (FLM) to study the quasi-3D static problems of simply supported, laminated piezoelectric plates, in which the plate was divided into a number of rectangular flat layers, and the trigonometric functions and Lagrange polynomials were used to interpolate the in- and out-of-plane variations of elastic displacements and electric potential of each individual layer, respectively. It has been demonstrated that this semi-analytical PVD-based FLM is more effective in reducing the computational effort and core requirements for simply supported plates. This FLM was also extended to the 3D static, vibration, stability and thermal buckling analyses of multilayered hybrid piezoelectric and elastic plates by Akhras and Li [58–60]. Based on the RMVT, rather than PVD, Wu and Li [61,62] developed a unified formulation of the FLMs for the quasi-3D static and free vibration analyses of multilayered composite and FGM plates, in which the material properties of each individual FGM layer were assumed to obey either an exponent-law exponentially varying with the thickness coordinate or an power-law distribution of the volume fractions of the constituents.

In this article, a unified formulation of PVD-based finite cylindrical layer methods (FCLMs) is developed for the quasi-3D bending and free-vibration analyses of simply-supported, sandwich circular hollow cylinders embedded in an FGM layer, in which an *h*-refinement process is adopted to achieve the convergent solutions such that the relative orders used for expansion of the displacement components remain variable, and can be freely chosen as linear, quadratic and cubic ones. The attention will be focused on the interface transverse stress reduction and continuous distribution of the in-surface stresses when the homogeneous core-layer in conventional sandwich cylinders is replaced by an FGM one. A comparison among the results obtained from these PVD-based FCLMs with different orders is presented. Moreover, a parametric study of the effects of the FGM core-layer, material-property gradient index, span- and thickness-to-middle surface radius ratios, and thickness ratio of each layer on the displacement and stress components induced in these sandwich FGM cylinders in the bending cases as well as the natural frequencies in the free vibration cases is carried out.

2. PVD-based finite cylindrical layer methods

2.1. Kinematic assumptions

We consider a simply supported, multilayered composite or FGM circular hollow cylinder, subjected to a sinusoidally distributed load on the outer or inner surface, as shown in Fig. 1. A global cylindrical coordinate system (x, θ and r coordinates) is located on the center of the cylinder, and a set of local thickness coordinates, z_m ($m = 1,2,3,\ldots,N_l$), is located at the mid-surface of each individual cylindrical layer, as shown in Fig. 1, where N_l is the total number of the layers constituting the cylinder, and is taken to be 3 for sandwich FGM cylinders. The thicknesses of each individual layer and the cylinder are h_m ($m = 1,2,\ldots,N_l$) and h, respectively, and $h = \sum_{m=1}^{N_l} h_m \cdot R$ and L denote the mid-surface radius and length of the cylinder, respectively. The relationship between the global and local thickness coordinates in the mth-layer is $\zeta = \bar{\zeta}_m + z_m$, in which $\bar{\zeta}_m = (\zeta_m + \zeta_{m-1})/2$, and ζ_m and ζ_{m-1} are the global thickness coordinates measured from the mid-surface of the cylinder to the top and bottom surfaces of the mth-layer, respectively.