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A mixed priority retrial queue with negative arrivals, unreliable server and multiple vacations

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ABSTRACT

A retrial queue accepting two types of positive customers and negative arrivals, mixed priorities, unreliable server and multiple vacations is considered. In case of blocking the first type customers can be queued whereas the second type customers leave the system and try their luck again after a random time period. When a first type customer arrives during the service of a second type customer, he either pushes the customer in service in orbit (preemptive) or he joins the queue waiting to be served (non-preemptive). Moreover negative arrivals eliminate the customer in service and cause server's abnormal breakdown, while in addition normal breakdowns may also occur. In both cases the server is sent immediately for repair. When, upon a service or repair completion, the server finds no first type customers waiting in queue remains idle and activates a timer. If timer expires before an arrival of a positive customer the server departs for multiple vacations. For such a system the stability conditions and the system state probabilities are investigated both in a transient and in a steady state. A stochastic decomposition result is also presented. Interesting applications are also discussed. Numerical results are finally obtained and used to investigate system performance.

1. Introduction

Queueing models with negative customers, or *G*-queues, were first introduced by Gelenbe [1,2] and have been found useful to model multiprocessor computer systems, neural networks, communication systems and manufacturing settings. In its simplest version, a negative customer have the effect of deleting a positive (ordinary) customer from the system, according to some strategy. For example, (i) arrival of a negative customer eliminates all the customers in the system (disasters), (ii) arrival of a negative customer that remove the customer from the head of queue, including the one in service, (iii) arrival of a negative customer that delete the customer at the end of queue. Negative arrivals have been interpreted as virus, orders or inhibitor signals. For a detailed study in queueing models with negative arrivals the reader is referred to [1,3–8] and to the monograph by Chao et al. [9].

Retrial queues are characterized by the feature that arriving customers who find all servers busy join the retrial group to try their luck again after a time period. Queues in which customers are allowed to conduct retrials have been extensively used to model many problems in telephone switching systems, telecommunication networks and computer systems for competing to gain service from a central processor unit. For a complete survey on this topic we refer [10–12] and the monographs by Falin and Templeton [13] and Artalejo and Gomez-Corral [14].

Several authors have studied retrial queues with priorities. High priority customers are queued and served according to some discipline. In case of blocking, low priority customers leave the system and retry until they find the server free. In

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related bibliography [15–17], the high priority customers have either preemptive or non-preemptive priority over the low priority customers. Moreover, in a paper by Artalejo et al. [18] repeated demands appeared to have preemptive priority over the waiting line.

In the last fifteen years, several papers deal with the queueing modelling of systems operating in the presence both of negative customers and repeated attempts. For a related bibliography we refer the papers by Artalejo and Gomez-Corral [19,20], Anisimov and Artalejo [21], Shin [22] and Liu et al. [23]. The above mentioned papers deals with continues time queueing models with one type of positive customers.

In most of the queueing literature the server is assumed to be reliable and always available to customers, but it is clear that this assumption in real systems such as communications and manufacturing systems, seems to be unrealistic. Retrial queues with server's breakdowns and repairs have been studied in several papers. As a related work we refer [24–26]. Moreover Wang et al. [27], also studied an active breakdown model from the viewpoint of reliability.

Several authors have analyzed so far queueing models with vacations in various combinations. For a complete survey on this topic see Doshi [28]. Moreover retrial queues with the concept of vacations are also widely studied. For a related bibliography see [17,29–31,23,32]. In all above mentioned papers are considered various vacation policies (single vacation, or Bernoulli or N policy or at most J vacations).

2. Contribution and applicability

In this paper we consider a retrial queueing model accepting two types of positive customers and negative arrivals, with the additional characteristics of mixed priorities, server's breakdowns and multiple vacations. The priority discipline that governs the service process is of mixed type.

More precisely, if a first type customer arrive during the service of a second type customer, the server either preempts the service of the second type customer and starts serving the newly arriving first type customer (preemptive priority), or the arriving first type customer is queued up waiting to be served (non-preemptive priority). In our opinion it is more realistic, in telephone switching systems or in communication systems the system manager may or not interrupts a call or a transmission of a message of low priority by the arrival of a higher priority one. For example, consider that high urgent calls may interrupt regular calls, while typical urgent calls do not interrupt regular calls.

Furthermore there are two kinds of failures in our model. When a negative customer arrives not only eliminates the customer in service but also cause a server's failure, called "abnormal breakdown" . Models with this behaviour of a negative arrival can be used to analyse computer networks with virus affection and breakdowns due to a reset order. The "normal breakdown" is due to inherent server's lifetime. In both cases the server is immediately sent for repair. Moreover, when the server either finishes all priority tasks or upon a repair completion there are no priority tasks waiting for service, the server activates a timer and remains idle awaiting the next customer that requests service. If the timer expires, the server departs for multiple vacations. With the later vacation policy the server is continuosly deals with secondary jobs, until high priority jobs arrive and request service.

To the author's best knowledge is the first time in the retrial literature that the concepts of mixed priorities among two types of positive customers, negative customers, timers and multiple vacation policy is combined.

In this work we try to fill the gap in the retrial queueing literature introducing mixed priorities that combines the disciplines of non-preemptive and preemptive priorities. Retrial queueing literature with priorities deals either with preemptive, or with non-preemptive priorities. In the model under consideration we combine both disciplines. Therefore many works on retrial queues with priorities become special cases of the proposed model.

We also assume for the first time in bibliography on retrial queues, the server's idle period to be limited by a timer. In related literature the server remains idle (and patient) waiting for a request. In this work the server become impatient and departs for vacation if the timer expires. In such a case the server become flexible and able to execute different jobs when there are no customer waiting in the priority queue.

Moreover the concept of timers allow the server to depart for multiple vacations. To the author's best knowledge it is the first time in related literature that the concept of multiple vacation in retrial queues is taken under consideration. The above mentioned characteristics are combined with the concepts of negative arrivals and server's breakdowns with repairs.

Besides its theoretical value, the system under consideration has interesting applications. In the sequel we present some of them.

Consider a computer network consisting of a group of processors that are connected with a central transmission unit. If a processor needs to send a message it sends it first to the central unit. The messages to be sent is of two types, urgent and regular. If the transmission medium (i.e., a bus in the engineering terminology) is unavailable, the urgent messages are stored in a buffer (ordinary queue), while the regular must be retransmitted (retrial) some time later. On the other hand if the medium is available the central unit sends immediately the message. Clearly urgent messages must be sent immediately and so, the medium give priority to the transmission of urgent over the regular messages. More precisely the system manager, must decide to preempt or not the transmission of a regular message when an urgent message arrives (mixed priority). Clearly, such systems are exposed to virus infection. When a virus enters the system it can destroy the message under transmission (negative arrival) and forces the system manager to make a reset of the system (repair from abnormal failure). Moreover mechanical parts of the system may breakdown, and cause a system failure that must be repaired (repair from normal failure). In such