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The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe: Analytical solutions

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ABSTRACT

This article examines the magnetohydrodynamic (MHD) flow of non-Newtonian nanofluid in a pipe. The temperature of the pipe is assumed to be higher than the temperature of the fluid. In particular two temperature dependent viscosity models, have been considered. The nonlinear partial differential equations along with the boundary conditions are first cast into a dimensionless form and then the equations are solved by homotopy analysis method (HAM). Explicit analytical expressions for the velocity field, the temperature distribution and nano concentration have been derived analytically. The effects of various physical parameters on velocity, temperature and nano concentration are discussed by using graphical approach.

1. Introduction

Magnetohydrodynamic (MHD) deals with the motion of conducting fluids. The applications of MHD cover a wide range of physical areas from liquid metals to cosmic plasmas; for instance, MHD pumps, MHD power generators, electrostatic precipitation, petroleum industry, electrostatic precipitation, purification of crude oil, aerodynamics heating, geophysics, plasma physics and fluid droplets sprays [1–6]. Moreover, non-Newtonian fluids [7–13] have been found much important and useful for technological point of view such as multi-grade oils, liquid detergents, paints, polymer solutions and polymer melts. Furthermore, recent advances in nanotechnology have led to the development of a new innovative class of heat transfer called nanofluids created by dispersing nanoparticles [14]. Non-Newtonian nanofluids are widely encountered in many industrial and technology applications, for example, melts of polymers, biological solutions, paints, tars, asphalts and glues etc. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas such as industrial cooling applications, nuclear reactors, transportation industry, micro-electromechanical systems, electronics and instrumentation, and biomedical applications. Nanofluid has also been found to possess enhanced thermophysical properties such as thermal conductivity, thermal diffusively, viscosity and convective heat transfer coefficients compared to those of base fluids like oil or water. A careful review of the literature reveals that a very little efforts are devoted to examine the non Newtonian nanofluid. Some relevant studies on the topic can be found from the list of Refs. [15,16].

Motivated by these facts, in the present study we have investigated the effects of MHD and variable viscosities on non-Newtonian nanofluid in a pipe. The flow is generated by constant pressure gradient. To derive the solutions of nonlinear

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governing equations, we have used an efficient method, homotopy analysis method (HAM) [17–21], which is particularly suitable for strongly nonlinear problems. After the introduction in Section 1, the outlines of this paper are as follows. Section 2 contains mathematical formulation. In Section 3 solutions of the problems are presented by using HAM. Convergence and discussion are given in Sections 4 and 5 respectively. Finally Section 6 summaries the concluding remarks.

2. Formulation of the problem

The governing equations of the fluid motion are the conservation of momentum

$$\rho \frac{d\mathbf{V}}{dt} = \mathrm{div}\,\mathbf{T} + \rho \mathbf{b} + \mathbf{J} \times \mathbf{B}, \tag{1}$$

where ρ is the density, d/dt is the material time derivative, $\mathbf{V}$ is the velocity field, $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{J}$ is the electric current density, $\mathbf{B}$ is the total magnetic field, $\mathbf{b} = -\rho\tilde{g}\mathbf{k}$, is the body force, $\mathbf{k}$ being the unit vector in the z-direction, and $\tilde{g}$ the acceleration due to gravity. The fact that the fluid undergoes only isochoric motion, therefore, the law of conservation of mass is defined by

$$\mathrm{div}\,\mathbf{V} = 0. \tag{2}$$

In view of the principle of conservation of heat energy, the energy equation for nanofluid is given by

$$\rho \frac{d\mathbf{e}}{dt} = \mathrm{div}\,\mathbf{Q} - (\rho c)_p \left[D_b \nabla \boldsymbol{\varphi} \cdot \nabla \boldsymbol{\theta} + \frac{D_t}{\theta} \nabla \boldsymbol{\theta} \cdot \nabla \boldsymbol{\theta} \right], \tag{3}$$

where $\mathbf{e}$ is specific internal energy, θ is temperature, c_p is specific heat, D_b is Brownian diffusion coefficient, D_t is thermophoretic diffusion coefficient and $\mathbf{Q}$ is heat flux.

According to Fourier's law of heat transfer

$$\mathbf{Q} = -k\,\mathrm{grad}\,\theta \tag{4}$$

and

$$\mathrm{div}\,\mathbf{Q} = -k \nabla \cdot (\nabla \theta), \tag{5}$$

k is thermal conductivity.

Due to complexity of non-Newtonian nanofluids, there is no single model which describes all of their properties. Therefore, several constitutive equations have been proposed which can describe all the behaviors of non-Newtonian nanofluids; for example, stress differences, shear thinning or shear thickening, stress relaxation, elastic effects and memory effects. Amongst the many models, there is a grade three model which is the most popular. This is particularly due to the fact that one can reasonably explain the shear thinning/shear thickening properties even for steady and unidirectional flows. The stress in a third grade fluid is given by

$$\mathbf{T} = -p\mathbf{I} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2 + \beta_1 \mathbf{A}_3 + \beta_2 (\mathbf{A}_1 \mathbf{A}_2 + \mathbf{A}_2 \mathbf{A}_1) + \beta_3 (tr\mathbf{A}_1^2)\mathbf{A}_1, \tag{6}$$

where μ is the coefficient of viscosity, p is hydrostatic pressure, $\mathbf{T}$ is Cauchy stress tensor, $-p\mathbf{I}$ is the spherical stress due to the constraint of incompressibility, α_i $(i = 1, 2)$ are material constants, β_j $(j = 1, 2)$ are grade three parameters and first three *Rivlin–Ericksen* kinematical tensors $\mathbf{A}_1, \mathbf{A}_2$ and $\mathbf{A}_3$ are defined by

$$\mathbf{A}_1 = (\mathrm{grad}\,\mathbf{V}) + (\mathrm{grad}\,\mathbf{V})^t, \tag{7}$$

$$\mathbf{A_n} = \frac{d\mathbf{A}_{n-1}}{dt} + \mathbf{A}_{n-1}(\mathrm{grad}\,\mathbf{V}) + (\mathrm{grad}\,\mathbf{V})^t A_{n-1}, \quad \text{for } n > 1, \tag{8}$$

where $\mathbf{V} = [0, 0, v(r)]$ denotes the velocity vector. If all the motions of the fluid are to be compatible with thermodynamics in the sense that these motions satisfy the Clausius–Duhem inequality and if it is assumed that the specific Helmholtz free energy is a minimum when the fluid is locally at rest, then thermodynamics imposes the following constraints [22]

$$\mu \geqslant 0, \alpha_1 \geqslant 0, \quad |\alpha_1 + \alpha_2| \leqslant \sqrt{24\mu\beta_3}, \quad \beta_1 = \beta_2 = 0, \quad \beta_3 \geqslant 0. \tag{9}$$

It is noted that this constitutive relation not only predicts the normal stress differences, but can also predict the "shear-thickening" phenomenon (since $\beta_3 > 0$) which is the increase in viscosity with increasing shear rate. In the present analysis we assume that the fluid is thermodynamically compatible, and therefore, Eq. (6) reduces to

$$\mathbf{T} = -p_1 \mathbf{I} + \left[\mu + \beta_3 (tr\mathbf{A}_1^2) \right] \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2. \tag{10}$$