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Robust stability criteria for a class of uncertain discrete-time systems with time-varying delay

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ABSTRACT

In this paper, we consider the problem of delay-dependent robust stability of a class of uncertain discrete-time systems with time-varying delay using Lyapunov functional approach. Two categories of time-varying uncertainties are considered for the robust stability analysis: viz., (i) nonlinear perturbations and (ii) norm-bounded uncertainties. In the proposed stability analysis, by exploiting a candidate Lyapunov functional, and using minimal number of slack matrix variables, less conservative stability criteria are developed in terms of linear matrix inequalities (LMIs) for computing the maximum allowable bound of the delay-range, within which, the uncertain system under consideration remains asymptotically stable in the sense of Lyapunov. The effectiveness of the proposed stability criteria is demonstrated using standard numerical examples.

1. Introduction

During the past two decades, considerable attention has been devoted by the control community in devising techniques for ascertaining stability of dynamical systems with time-delays; refer [1–3], and the references cited therein. Time-delays are associated in various physical systems like communication systems, air-craft stabilization, nuclear reactors, population dynamics, ship stabilization and electric power systems with lossless transmission lines, etc.; these delays are time-varying in nature, and their presence in a system has an adverse impact not only on system performance, but also on its stability. Depending upon whether or not the stability criteria for a time-delay system contains the information of time-delay, the criteria can be classified respectively into two categories: namely, the delay-dependent stability criteria and delay-independent stability criteria. Since delay-dependent criteria make use of information on the length of the time-delay, they are less conservative than the delay-independent ones. Hence, researchers have focussed on the delay-dependent stability problem of discrete-time system with time-delay, and many significant results have been reported in the recent past for ascertaining the delay-dependent stability of discrete-time systems with time-varying delay, notable among them being [4–11]. Among these results [10,11] are recently reported ones; [10] uses free-weighting matrices approach, and [11] uses reciprocal convex approach. However, the results [4–11] does not take into account the presence of nonlinear perturbations in the system.

In practice, owing to presence of environmental noise, uncertain or slowly varying parameters etc., discrete-time systems may be subjected to time-varying nonlinear perturbations. Under such conditions, the results presented in [4–11] cannot be employed for ascertaining delay-dependent stability. For continuous-time systems with time-varying delay and nonlinear perturbations, there are quite a number of useful results presented in [12–20]. For discrete-time systems with nonlinear perturbations, delay-dependent stability criteria are presented in [21]; however, in the robust stability analysis, neglecting

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certain useful terms leads to conservativeness of the resulting criterion (see, Remark 1), and offers motivation for further investigation.

In this paper, we consider the delay-dependent robust stability problem of a class of uncertain discrete-time system with time-varying delay, and nonlinear perturbations using Lyapunov functional approach. Subsequently, by exploiting a candidate Lyapunov functional, and using minimal number of slack matrix variables in the delay-dependent stability analysis, a less conservative robust stability criterion is derived in LMI framework. In the sequel, norm-bounded uncertainties is considered as a special case, and the effectiveness of the proposed stability criterion is validated mathematically against a recently reported result [9]. Finally, numerical examples are employed to demonstrate the effectiveness of the proposed results.

Notations. Notations used in this paper are fairly standard: $\mathbb{R}^n$ denotes the n-dimensional Euclidian space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, I and 0 represents the identity matrix and null matrix of appropriate dimensions; the superscript T stands for the matrix transposition; $X > 0$ (respectively $X \geqslant 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix is real symmetric positive definite (respectively, positive semi definite); $\mathbb{R}(\mathbb{Z})$ denotes the set of real numbers (integers). The symbol '$\bigstar$' represents the symmetric elements in a symmetric matrix.

2. System description and problem formulation

Consider the uncertain discrete-time system:

$$\left.\begin{aligned} &x(k+1) = Ax(k) + A_d x(k-d(k)) + f(x(k),k) + g(x(k-d(k)),k), \\ &x(k) = \Phi(k), \quad k = -d_2, -d_2+1, \ldots, 0, \end{aligned}\right\} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; A and A_d are known real constant matrices of appropriate dimensions; $f(x(k),k)$ and $g(x(k-d(k)),k)$ are respectively nonlinear perturbations in the current and the delayed state. They are described by

$$f^T(x(k),k)f(x(k),k) \leqslant \alpha^2 x^T(k)F^TFx(k), \tag{2}$$

$$g^T(x(k-d(k)),k)g(x(k-d(k)),k) \leqslant \beta^2 x^T(k-d(k))G^TGx(k-d(k)), \tag{3}$$

where F and G are known constant matrices, and $\alpha \geqslant 0$ and $\beta \geqslant 0$ are known scalars. The sequence $\Phi(k)$ is the initial condition; the time-varying delay $d(k)$ satisfies the following condition:

$$0 < d_1 \leqslant d(k) \leqslant d_2, \tag{4}$$

where d_1 and d_2 are non-negative integers representing the lower and upper bounds of the interval time-delay. It is assumed that $d_1 \neq d_2$; hence, $d_{12} = (d_2 - d_1) \neq 0$. As a special case, we consider the parametric uncertainties as $f(x(k),k) = \Delta A(k)$ and $g(x(k-d(k)),k) = \Delta A_d(k)$, in which case, the system descriptive equation becomes:

$$\left.\begin{aligned} &x(k+1) = (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k))x(k-d(k)), \\ &x(k) = \Phi(k), \quad k = -d_2, -d_2+1, \ldots, 0, \end{aligned}\right\} \tag{5}$$

wherein, the parametric uncertainties $\Delta A(k)$ and $\Delta A_d(k)$ are assumed to be norm-bounded of the form:

$$[\,\Delta A(k) \quad \Delta A_d(k)\,] = G\Delta(k)[\,H \quad H_d\,], \tag{6}$$

where G, H and H_d are real known constant matrices of appropriate dimensions, and $\Delta(k)$ is a real unknown time-varying matrix with Lebesgue measurable elements satisfying

$$\Delta^T(k)\Delta(k) \leqslant I. \tag{7}$$

This paper investigates the delay-dependent robust stability of the uncertain discrete-time systems (1) and (5) satisfying the time-varying delay (4), and formulate less conservative stability criteria (sufficient conditions) in LMI framework for estimating the maximum allowable bound for the delay-range $[d_1, d_2]$, within which, the systems remain asymptotically stable in the sense of Lyapunov. Following lemmas are indispensable in deriving the proposed stability criteria:

Lemma 1 ([9] Discrete Jenson inequality). *For any constant matrix $W \in \mathbb{R}^{n\times n}$ with $W = W^T > 0$, integers $l_1 < l_2$, vector function $\omega : \{l_{1,1}+1, \ldots, l_2\} \mapsto \mathbb{R}^m$ such that the sums concerned are well defined, then*

$$(l_2 - l_1 + 1)\sum_{i=l_1}^{l_2} \omega^T(i)W\omega(i) \geqslant \left(\sum_{i=l_1}^{l_2}\omega(i)\right)^T W \left(\sum_{i=l_1}^{l_2}\omega(i)\right). \tag{8}$$

Lemma 2 (*Discrete Newton–Leibniz formula*). *If $u(i+1) - u(i) = t(i)$, then*

$$\sum_{i=v}^{w} t(i) = u(w+1) - u(v). \tag{9}$$