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Structural optimization of an acoustic horn

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ABSTRACT

The objective of the present article is to find an optimal design of an acoustic horn in the case that the magnitude of the reflection wave integrated over the inflow boundary is to be minimized meanwhile the Helmholtz equation models the wave propagation. In contrast to the current approaches such as gradient-based optimization algorithms, we employ here a non-iterative method based on measure theory which dose not require any information of gradients and the differentiability of objective function in the optimization problem is not as a rule. Implementation of our fast convergence approach shows that the resulting horns, not only for single frequency optimization but also for a band of frequencies, are very efficient.

1. Introduction

Traditionally, design optimization is an interested subject and challenging issue in engineering design and industrial applications which has captured the minds of researchers for many years. In most of literature, which is not the case in this study, design optimization is a generate-and-test procedure that executed iteratively by comparing various solutions. The most common way of finding an optimal design is based on parametrization of the geometry and then the optimal design can be explored in the parameter space. Since the geometry is generally parameterized from a given *reference configuration*, the results become affected essentially by the choice of parametrization. In the context of design optimization the two branches *boundary shape optimization* and *topology optimization* have been more developed than the others. In boundary shape optimization [1–3] in order to examine changes of geometry, boundary displacements of a reference configuration are considered. Topology optimization [4–7] is carried out the same as in shape optimization but for a larger class of feasible structural domains and the aim is to find the optimal material distribution in a specified region.

The concern of this paper is to study and apply shape optimization to an acoustic horn to transmit an incoming wave as efficiently as possible. In order to design an acoustic horn, we consider precisely the same setup as in [1], but instead of a quasi-Newton algorithm, we employ an innovative approach called measure theoretical (MT) to perform the shape optimization. The main difficulties in gradient-based optimization algorithms, for instance, the modified quasi-Newton algorithm implemented in [1], are: (i) It is required to compute the gradient of the objective function with respect to the model parameters and they are likely to work well only, if the objective functional is differentiable. Namely, it seems that they cannot enable to tackle the non-differentiability problem. (ii) To supply gradient information to such algorithms, the most widely-used method is to solve an *adjoint equation* in addition to the *state equation*. (iii) The terms smoothing and regularization are generally employed in a naive implementation of gradient-based optimization algorithms to provide an efficient cure to the problems: *existence of solutions*, *wiggly shapes* and *local minima*. Such additional requirements actually impose extra constraints such as Poisson problem on the optimization problem and make computation more expensive. (iv) In multi frequency optimization, the proposed procedure in [1] depends strictly on the use of smoothing to avoid mesh corruption.

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Furthermore, the latter method bears an important drawback which is that the resulting shapes depend heavily on the initial ones.

Because of these issues, we have been encouraged to apply MT approach for the shape optimization problem was investigated in [1]. The MT approach, inspired of the detailed work [8], has been extended successfully to optimal control problems [9–12] and optimal shape design problems [13–15]. The use of MT approach allows us to carry out optimization in any coarse model space with regards to: (i) The design process is not iterative and therefore requires no initial design to be suggested and evaluated. (ii) The MT algorithm is computationally efficient and flexible enough to accommodate general design problems. (iii) The proposed approach here is computationally advantageous for boundary shape design problems, this is because, further a set of extra constraints such as adjoint equations and Poisson problem are not imposed on the model building within MT scheme. (iv) The interesting use of MT approach is to optimize the topology without concerning on the differentiability of the objective function and is to generate global minima which is numerically close to what one could reasonably call the global infimum of the optimization problem. Notice that the latter issue is related to our framework built later.

In the work described hereafter, the proposed approach is based on the injection of the classical admissible vectors, particularly trajectory–control pairs, into a space of measures. In doing so, the classical problem is replaced by a measure-theoretical one in which it is desirable to minimize a linear form over a subset of space of all positive Radon measures determined by the constraints of the problem. As a matter of fact, the latter subset could be described by linear equalities and of course the whole machinery of linear analysis can be used to attack nonlinear problems. Moreover, the existence of an optimal measure minimizing a linear form over a set of Radon measures can be studied in a straightforward manner without having to impose conditions such as convexity.

The main parts of mechanism through which the method and structure of the paper may be configured, deserve brief description here:

Step 1. Among MT implementation, it is necessary that a fixed geometry be considered. To do this task efficiently, a bijective transformation is defined to equivalent the varied geometry of the planar contraction into a fixed one.
Step 2. Any admissible shape is replaced by exactly one point (triple) in a geometry and then an injective mapping corresponds this point to a pair of positive, linear and continuous functionals.
Step 3. The process continues by establishing a minimization problem of a linear form over a set of pairs of non-negative Radon measures satisfying linear constraints.
Step 4. The minimization in the mentioned problem is global, but in order to treat computationally, the reduced problem in measure space is approximated by a finite-dimensional linear programming problem.

In the approximation process, the optimal measure is approximated by a finite combination of unitary atomic measures. The optimal solution of the finite-dimensional linear programming problem, which approximates that of the original shape optimization problem, is used to construct an optimal piecewise constant control. Then, the approximate optimal shapes are obtained by the aid of the approximate optimal control signals.

Finally, the detailed proof of the main results which ignored in the paper can be found in the Appendix.

2. Problem statement

The geometry of interest is the planar symmetric *horn* shown in Fig. 1. The planar channel, the *waveguide*, has infinite extension to the left. Geometry of horn is in fact the termination of the waveguide. We assume that the wave propagates from left to right in the waveguide with amplitude A. Since MT approach applied to solve the wave propagation problem

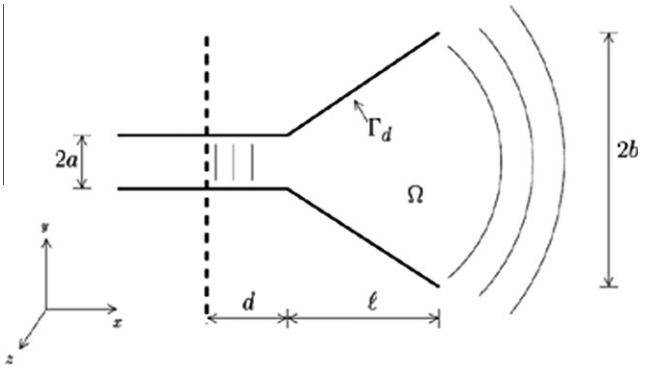

Fig. 1. Geometry of the acoustical horn.