Applied Mathematical Modelling 34 (2010) 4138–4150

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An optimal replacement policy for a multistate degenerative simple system ☆

Yuan Lin Zhang *, Guan Jun Wang

Department of Mathematics, Southeast University, Nanjing 210096, PR China

ARTICLE INFO

Article history:
Received 5 July 2009
Received in revised form 3 April 2010
Accepted 22 April 2010
Available online 29 April 2010

Keywords:
Geometric process
General monotone process
Replacement policy T
Renewal reward theorem
Harmonic mean

ABSTRACT

In this paper, a degenerative simple system (i.e. a degenerative one-component system with one repairman) with $k+1$ states, including k failure states and one working state, is studied. Assume that the system after repair is not "as good as new", and the degeneration of the system is stochastic. Under these assumptions, we consider a new replacement policy T based on the system age. Our problem is to determine an optimal replacement policy T^* such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. The explicit expression of the average cost rate is derived, the corresponding optimal replacement policy can be determined, the explicit expression of the minimum of the average cost rate can be found and under some mild conditions the existence and uniqueness of the optimal policy T^* can be proved, too. Further, we can show that the repair model for the multistate system in this paper forms a general monotone process repair model which includes the geometric process repair model as a special case. We can also show that the repair model in the paper is equivalent to a geometric process repair model for a two-state degenerative simple system in the sense that they have the same average cost rate and the same optimal policy. Finally, a numerical example is given to illustrate the theoretical results of this model.

1. Introduction

At the earlier stage, many replacement models for a simple repairable system assumed that the system after repair is "as good as new". This is a perfect repair model. However, it is not always true in practice. In other words, the system after repair cannot be "as good as new". Under this assumption, Barlow and Hunter [1] first presented a minimal repair model in which the minimal repair does not change the age of the system, i.e. the system after repair has the same failure rate and the same effective age as at the time of failure. Later on, Brown and Proschan [2] first considered an imperfect repair model in which the repair will be perfect repair with probability p or minimal repair with probability $1-p$. Many research works have been done by Block et al. [3], Kijima [4], Makis and Jardine [5], Sheu [6], Sheu and Chien [7] and others along this direction.

However, for a degenerative simple system, it seems more reasonable to assume that the successive working times of the system after repair will become shorter and shorter, while the consecutive repair times of the system will become longer and longer. Ultimately, it cannot work any longer, neither can it be repaired. In other words, the successive working times of the system are stochastically decreasing, while the consecutive repair times of the system are stochastically increasing. To model such a degenerative system, Lam [8,9] first introduced a geometric process repair model in which he studied two kinds of

☆ This work was fully supported by the National Natural Science Foundation of China under Grant No. 10801032.

* Corresponding author.
E-mail addresses: orylz@163.com (Y.L. Zhang), wgjmath@gmail.com (G.J. Wang).

0307-904X/$ - see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2010.04.011

replacement policy, one based on the working age T of the system and the other based on the failure number N of the system. The explicit expressions of the average cost rate under these two kinds of policy can be calculated. The objective is to choose optimal replacement policies T^* and N^*, respectively, such that the average cost rate is minimized. Because the geometric process is a special monotone process, Stadje and Zuckerman [10] introduced a general monotone process repair model to generalize Lam's work. Finkelstein [11] presented a general repair model based on a scale transformation after each repair to generalize Lam's work. Zhang [12] generalized Lam's work by a bivariate replacement policy (T,N) under which the system is replaced at the working age T or at the time of the Nth failure, whichever occurs first. Zhang [13] applied the geometric process repair model to a two-component cold standby repairable system with one repairman and considered a replacement policy N based on the repair number of component 1. The problem is to determine the optimal replacement policy N^* such that the long-run expected reward per unit time is maximized. Further, Zhang and Wang [14] applied the geometric process repair model to a deteriorating two-component cold standby repairable system with priority in use, they not only studied some important reliability indices, but also considered a replacement policy based on the working age T of component 1. An optimal replacement policy T^* for minimizing the average cost rate of the system can be found. Many research works on the geometric process repair model have been done by Stadje and Zuckerman [15], Lam [16], Stanley [17], Lam and Zhang [18–21], Zhang [22,23], Wang and Zhang [24–28], Zhang and Wang [29–31] and Zhang and Wu [32].

In most existing models for maintenance problems, including the geometric process repair models, it is usually assumed that a system has only two states: a working state and a failure state. However, in many practical situations, a system may be a multistate system with more than two states. For example, the mechanical component in a system may be divided into slight failure and serious failure, the switch component such as relay and diode in a system may be divided into short circuit failure and open circuit failure, the automatic control component in a system may be divided into safe failure and dangerous failure, the block component in a system may be divided into interval failure and permanent failure and etc. Thus, Zhang et al. [33] studied a degenerative simple system with $k+1$ states, including k failure states and one working state. When the system after repair cannot be "as good as new", a replacement policy N based on the number of failures of the system is applied. The objective is to maximize the long-run expected profit per unit time. The explicit expression of the long-run expected profit per unit time is derived and the corresponding optimal solution may be determined analytically or numerically. Furthermore, it is showed that the repair model for the multistate degenerative simple system forms a general monotone process repair model which includes the geometric process repair model as a special case. See also Lam et al. [34], Zhang [35] and Zhang et al. [36] for other researches.

In the all above research works for the simple repairable systems, including the multistate simple repairable systems, the replacement policy is either T based on the working age of the system or N based on the failure (or repair) number of the system. The objective of this paper is to study a maintenance model for a multistate degenerative simple system. Assume that the system after repair cannot be "as good as new", and the degeneration of the system is stochastic. Under these assumptions, we consider a new replacement policy T based on the system age. The system age is the calendar age of the system, which is very easy to measure. The models reported by Sheu [6], Sheu and Chien [7], Lam [9], Zhang [12], Zhang and Wang [14], Zhang et al. [36] and etc. used the working age of the system. The working age is actually a cumulative time when the system is in working state. Obviously, if the system has to make replacement or repair decision based on the working age, it requires extra effort to measure and record the actual working time of the system. Thus, the purpose of this paper is to suggest a convenient replacement policy from the managerial point of view. Our problem is then to determine an optimal replacement policy T^* based on the system age such that the objective function-the average cost rate of the system is minimized. The explicit expression of the average cost rate of the system is derived, and the corresponding optimal replacement policy can be determined. Under some mild conditions the existence and uniqueness of the optimal policy T^* can be proved, and the explicit expression of the minimum of the average cost rate of the system can be found. We can also prove that the repair model for the multistate system in this paper forms a general monotone process repair model which includes the geometric process repair model as a special case. Further, we can show that this model in the paper is equivalent to a geometric process repair model for a two-state degenerative simple system in the sense that they have the same objective function and the same optimal policy. Finally, a numerical example is given to illustrate the theoretical results of the model.

2. Definitions and assumptions

For easy of reference, we first state the definitions of stochastic order, geometric process and harmonic mean as follows.

Definition 1. Given two random variables ξ and η, ξ is said to be stochastically larger than η or η is stochastically smaller than ξ, if

$$P(\xi > \alpha) \geqslant P(\eta > \alpha), \quad \text{for all real } \alpha,$$

denoted by $\xi \geqslant_{st} \eta$ or $\eta \leqslant_{st} \xi$ (see e.g. Ross [37]). Furthermore, we say that a stochastic process $\{X_n, n = 1, 2, \ldots\}$ is stochastically decreasing if $X_n \geqslant_{st} X_{n+1}$ or stochastically increasing if $X_n \leqslant_{st} X_{n+1}$ for all $n = 1,2,\ldots$

Definition 2. A stochastic process $\{\xi_n, n = 1,2,\ldots\}$ is a geometric process, if there exists a real $a > 0$ such that $\{a^{n-1}\xi_n, n = 1, 2,\ldots\}$ forms a renewal process. The real a is called the ratio of the geometric process (see e.g. Lam [8,9] and Zhang [12] for more details).