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A new element for analyzing large deformation of thin Naghdi shell model. Part 1: Elastic

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ABSTRACT

One of the best approaches for modeling large deformation of shells is the Cosserat surface. However, the finite-element implementation of this model suffers from membrane and shear locking, especially for very thin shells. The basic assumption of this theory is that the mid-surface of the shell is regarded as a Cosserat surface with one inextensible director. In this paper, it is shown that by constraining the director vector normal to the mid-surface, besides very good and accurate results, shear locking is also eliminated. This constraint is in fact a limiting analysis of the Cosserat theory in which Kirichhoff's hypothesis is enforced. Numerical solution is performed using nine-node isoparametric element. The principal of virtual work is used to obtain the weak form of the governing differential equations and the material and geometric stiffness matrices are derived through a linearization process. The validity and the accuracy of the method are illustrated by numerical examples.

1. Introduction

Interesting in the large deformation response of plates and shells grew as a direct consequence of the use of thinner sections as a mean of achieving material economy. Analytical and numerical analysis of shell structures have been carried out for a very long time. Yang et al. [1,2] presented a survey of the development of the shell theory starting from the origination of the curved shell finite elements in the mid-1960s.

In finite-element approximation of shell models two distinct classes of shell elements emerge [3]:

(1) Degenerated shell elements based on three dimensional continuum theory.
(2) Shell elements founded on the classical shell theory.

The degenerate solid approach is described firstly in the paper of Ahmad et al. [4]. The works of Ramm [5], Bathe and Dvorkin [6], and Liu et al. [7] among many others, are representative of such an approach.

The second afore-mentioned methodology represents a return to the origins of classical shell theory, which has its modern point of departure in the pioneering work of Cosserat that further elaborated upon by a number of authors (Naghdi [8], Antman [9], Simo et al. [10–12]). The basic assumption of this theory is that the mid-surface of the shell is regarded as a Cosserat surface with one inextensible director. Typically this approach yields an exact analytical definition of the initial geometry of the shells and the stress and strain are represented in a curvilinear coordinate system.

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In the many of above-mentioned works, shear deformations in the direction of the thickness are taken into account. In the finite-element models, taking the transverse shear strain into account, for thin shells, causes an undesirable numerical effect, the so-called shear locking phenomenon. Consequently, as the thickness of the plate and shell becomes extremely thin, the shear strain energy predicted by the finite-element analysis can vary unreasonably, even though the average value of the shear strain over the area approaches zero.

Membrane locking phenomenon occurs due to different orders of magnitude for the membrane and bending strains when the shell is thin. In Cosserat theory, since director and displacement shape functions are different, membrane locking may also arise from inconsistency between the director and displacement shape functions. Therefore, the matching field approach and high-order shape functions need to be used to avoid this problem [3].

According to the well known Kirichhoff's hypothesis, straight lines perpendicular to the mid-surface remain perpendicular to the deformed mid-surface. This hypothesis yields satisfactory results only when the thickness approaches zero and the deformation is not large. This hypothesis can lead to numerical difficulties, if used for large deformations. However, it will be shown in this paper that by employing Cosserat's surface and constraining the director vector to be normal to the mid-surface, very good results can be obtained for large deformation of thin plates and shells. This constraint is in fact a limiting analysis of the Cosserat theory in which Kirichhoff's hypothesis is enforced, hence the shear strains in the direction of the shell's thickness are ignored and no shear locking can occur. Also by employing this idea, it does not require interpolating the director vector separately. Therefore, there is no concern about consistency between displacement and director shape functions and the speed of solution increases because only displacement field needs to be interpolated. In the other word advantages of Cosserat surface and Kirichhoff theory are collected together.

Numerical solution is performed using nine-node isoparametric element. The principal of virtual work is used to obtain the weak form of the governing differential equations and the material and geometric stiffness matrices are derived through a linearization process. The validity and the accuracy of the method are illustrated by numerical examples.

The outline of this paper is as follows. In Section 2 the theory is explained. In Section 3 the problem is proposed for nine-node isoparametric element and the stiffness matrices are derived. In Section 4 several numerical examples are presented and the results are compared with literature. Finally, conclusions are drawn in Section 5.

2. Theory

Fig. 1 shows geometry of a three dimensional shell with a mid-surface (M). On the mid-surface the convective coordinate system θ^1, θ^2 is considered which has the base vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ which is orthogonal to $\mathbf{a}_1$ and $\mathbf{a}_2$. The position vector of any point with respect to O is [13]:

$$\mathbf{R} = \mathbf{r}(\theta^1, \theta^2) + \theta^3 \mathbf{a}_3. \tag{1}$$

Fig. 2 shows the mid-surface of an arbitrary shell in equilibrium states before and after deformation ($t = 0, t$, respectively). In this figure x, y and z represent reference Cartesian coordinate system and θ^1, θ^2 are the convective coordinate system.

As shown in Fig. 2, the base vectors of the convective coordinate system in initial configuration are denoted by ${}^0\mathbf{a}_i$. Similarly, ${}^t\mathbf{a}_i$ denotes the base vectors of the convective coordinate system at time "t". It should be noted that the director vector is constrained to be perpendicular to the mid surface at each time; hence ${}^t\mathbf{a}_3 = {}^t\mathbf{d}$. The position vector of a material point, which is a function of θ^1 and θ^2, becomes:

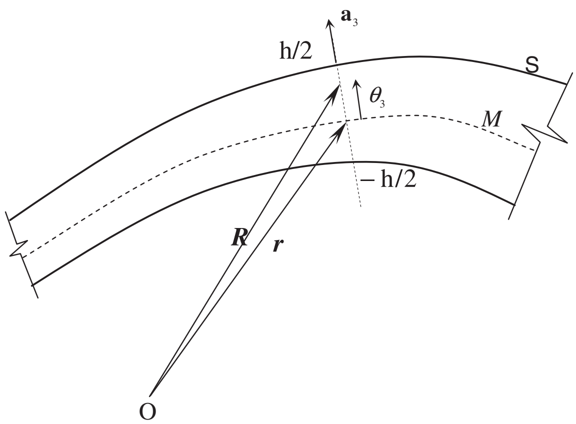

Fig. 1. Geometry of a three dimensional shell.