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# Analysis of flow and thermal field in nanofluid using a single phase thermal dispersion model

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ABSTRACT

Flow and thermal field in nanofluid is analyzed using single phase thermal dispersion model proposed by Xuan and Roetzel [Y. Xuan, W. Roetzel, Conceptions for heat transfer correlation of nanofluids, Int. J. Heat Mass Transfer 43 (2000) 3701–3707]. The non-dimensional form of the transport equations involving the thermal dispersion effect is solved numerically using semi-explicit finite volume solver in a collocated grid. Heat transfer augmentation for copper–water nanofluid is estimated in a thermally driven two-dimensional cavity. The thermo-physical properties of nanofluid are calculated involving contributions due to the base fluid and nanoparticles. The flow and heat transfer process in the cavity is analyzed using different thermo-physical models for the nanofluid available in literature. The influence of controlling parameters on convective recirculation and heat transfer augmentation induced in buoyancy driven cavity is estimated in detail. The controlling parameters considered for this study are Grashof number ($10^3 < Gr < 10^5$), solid volume fraction ($0 < \phi < 0.2$) and empirical shape factor ($0.5 < n < 6$). Simulations carried out with various thermo-physical models of the nanofluid show significant influence on thermal boundary layer thickness when the model incorporates the contribution of nanoparticles in the density as well as viscosity of nanofluid. Simulations incorporating the thermal dispersion model show increment in local thermal conductivity at locations with maximum velocity. The suspended particles increase the surface area and the heat transfer capacity of the fluid. As solid volume fraction increases, the effect is more pronounced. The average Nusselt number from the hot wall increases with the solid volume fraction. The boundary surface of nanoparticles and their chaotic movement greatly enhances the fluid heat conduction contribution. Considerable improvement in thermal conductivity is observed as a result of increase in the shape factor.



## 1. Introduction

Nanofluid is a suspension of nanoparticles in base fluid. Nanofluids have attracted enormous interest from researchers due to their potential for high rate of heat exchange incurring either little or no penalty in pressure drop. The convective heat transfer characteristic of nanofluids depends on the thermo-physical properties of the base fluid and the ultra fine particles, the flow pattern and flow structure, the volume fraction of the suspended particles, the dimensions and the shape of these particles. The utility of a particular nanofluid for a heat transfer application can be established by suitably modeling the convective transport in the nanofluid.

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**Nomenclature**

*Principal symbols*

| | |
|---|---|
| $A$ | Area |
| Gr | Grashof number $(g\beta_f \Delta T L^3/\nu_f^2)$ |
| $L$ | Height and width of domain (Characteristic dimension) |
| Nu | Nusselt number |
| $\overline{Nu}$ | Average Nusselt number |
| $P$ | Pressure, N m$^{-2}$ |
| Pr | Prandtl number $(\nu_f/\alpha_f)$ |
| $Q$ | Total heat transfer |
| $\mathbf{S}$ | Surface area vector, $m^2$ |
| $T$ | Temperature, $K$ |
| $c_p$ | Specific heat at constant pressure, J kg$^{-1}$ K$^{-1}$ |
| $d_p$ | Diameter of the nanoparticle, nm |
| $V$ | Control volume, m$^3$ |
| $g$ | Gravity acceleration, m$^2$ s$^{-1}$ |
| $k_f$ | Thermal conductivity of the fluid, W m$^{-1}$ K$^{-1}$ |
| $k_s$ | Thermal conductivity of the solid, W m$^{-1}$ K$^{-1}$ |
| $n$ | Empirical shape factor |
| $T$ | Time, second |
| $\boldsymbol{u}$ | Velocity vector, m$^2$ s$^{-1}$ |
| $u$, $v$ | Velocity component in coordinate directions, m s$^{-1}$ |
| $x$, $y$ | Distance in coordinate direction, m |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | Thermal diffusivity, m$^2$ s$^{-1}$ |
| $\beta_f$ | Fluid expansion coefficient |
| $\beta_s$ | Solid expansion coefficient |
| $\phi$ | Volume fraction of suspension particles |
| $\Psi$ | Sphericity |
| $\mu$ | Dynamic viscosity, N s m$^{-2}$ |
| $\upsilon$ | Kinematics viscosity, $\mu/\rho$, m$^2$ s$^{-1}$ |
| $\rho$ | Density, kg m$^{-3}$ |
| $\theta$ | Non-dimensional temperature |

*Subscripts*

| | |
|---|---|
| $f$ | Base fluid |
| $nf$ | Nanofluid |
| $s$ | Solid |
| $h$ | Hot wall |

*Superscript*

| | |
|---|---|
| * | Non-dimensional terms |

Several authors have tried to establish convective transport models for nanofluids. Nanofluid is a two-phase mixture in which the solid phase consists of nano-sized particles. In view of the nanoscale size of the particles, it may be questionable whether the theory of conventional two-phase flow can be applied in describing the flow characteristics of nanofluid [1]. Since the size of the particles is less than 100 nm, nanofluids behave like a fluid than a mixture [1–3]. Xuan and Roetzel [1] proposed homogeneous flow model where the convective transport equations of pure fluids are directly extended to nanofluids. This means that all traditional heat transfer correlations (e.g. Dittus–Boelter) could be used for nanofluids provided the properties of pure fluids are replaced by those of nanofluids involving the volume fraction of the nanoparticles. The homogeneous flow models are however in conflict with the experimental observations of Maliga et al. [3], as they under predict the heat transfer coefficient of nanofluids.

The main flow in nanofluid involves effect of gravity, Brownian force and friction force between the fluid and the ultrafine particles, the phenomena of Brownian diffusion, sedimentation and dispersion. Thus although the nanoparticles are ultrafine, the slip between the fluid and the particles may not be zero. The random and irregular movement of the particles increases the energy exchange rates in fluids. To account for this enhanced exchange rate Xuan and Roetzel [1] also proposed a single-phase thermal dispersion model. In this approach, the effect of nanoparticle/base fluid relative velocity is treated as a perturbation of the energy equation and an empirical dispersion coefficient is introduced to describe the heat transfer