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Influence of power-law index on transitional Reynolds numbers for flow over a semi-circular cylinder

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ABSTRACT

In this work, the governing partial differential equations (continuity and Cauchy's momentum equations) describing the flow of power-law type non-Newtonian fluids across a semi-circular cylinder (oriented with its curved surface in the upstream direction) have been solved numerically. In particular, consideration has been given to the delineation of the critical Reynolds numbers denoting the onset of flow separation from the surface of the cylinder and the onset of the laminar vortex shedding regime. This information is germane to establish the scaling of the macroscopic characteristics like drag coefficient and Strouhal number on the governing parameters, namely, Reynolds number and power-law index. The present results clearly suggest that the transitional Reynolds numbers show a strong dependence on the type (shear-thinning and shear-thickening) of fluid behavior as well as on the severity of the shear-dependence of the viscosity. With reference to the behavior seen in Newtonian fluids, the flow remains not only attached to the surface up to higher Reynolds numbers, but shear-thinning behavior also delays the onset of the laminar vortex shedding regime. As expected, shear-thickening fluids, of course, display the opposite characteristics.

1. Introduction

The flow over bluff bodies of various shapes constitutes an important branch of fluid mechanics and consequently, it has been studied well over 100 years. The interest in this class of problems stems from both pragmatic and theoretical considerations. For instance, the flow over a cylinder represents an idealization of many industrially important processes including the flow in pin-type and tubular heat exchangers, membrane-based separation modules, food processing applications, etc. On the other hand, such model flows also provide useful insights leading to an improved understanding of the underlying physical phenomena. Reliable knowledge of the detailed structure of the flow field, and the global characteristics, drag, wake phenomenon, heat transfer, etc., facilitate improved and efficient design of process equipment.

The flow of Newtonian fluids over variously shaped bluff bodies has been explored extensively and thus voluminous information has accrued [1–5] over the years, especially for cylinders of circular, elliptic, rectangular and square cross-sections oriented transverse to the direction of flow, although there is a preponderance of studies dealing with a single and collection of circular cylinders. Suffice it to say here that a reasonable body of knowledge also exists for cylinders of elliptic, square, rectangular and triangular shaped cylinders as far as the flow of Newtonian fluids is concerned. However, one of the much less studied shapes is semi-circular cylinder in various configurations which are encountered in novel heat exchangers, subsea vessels like submarines, continuous thermal processing of sliced carrots and potato chips, etc. in food

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Nomenclature

C_D	total drag coefficient
$\overline{C}_D$	time average drag coefficient
C_{DF}	friction drag coefficient
C_{DP}	pressure drag coefficient
C_L	total lift coefficient
$C_{L_{rms}}$	root mean square value of lift coefficient
C_p	pressure coefficient
D	diameter of cylinder (m)
D_∞	diameter of computational domain
e	aspect ratio of elliptic cylinder (dimension in x-direction/dimension in y-direction)
f	frequency of vortex shedding (s^{-1})
F_D	total drag force per unit length of cylinder (N/m)
F_{DF}	friction drag force per unit length of cylinder (N/m)
F_{DP}	pressure drag force per unit length of cylinder (N/m)
F_L	total lift force per unit length of cylinder (N/m)
m	power-law consistency index (Pa s^n)
n	power-law index
n_x	x-component of unit vector normal to the surface of the cylinder
n_s	unit vector normal to the surface of the cylinder
N	total number of cells
N_P	number of grid points on the surface of the semi-circular cylinder
p	local surface pressure (N/m^2)
p_∞	free stream pressure (N/m^2)
Re	Reynolds number
Re^c	critical Reynolds number for the onset of flow separation
Re_c	critical Reynolds number for the onset of vortex shedding
S	surface area (m^2)
St	Strouhal number
St_c	critical Strouhal number
U	velocity (m/s)
U_∞	free stream velocity at the inlet (m/s)
U_x	x-component of the velocity (m/s)
U_y	y-component of the velocity (m/s)

Greek symbols

ε	rate of deformation tensor (s^{-1})
δ	grid spacing in the vicinity of the cylinder (m)
φ	U_x and U_y
η	viscosity (Pa s)
ρ	density of the fluid (kg/m^3)
τ	extra stress tensor (Pa)

Subscripts

i, j, x, y, z	cartesian coordinate

Superscript

$'$	dimensional variable

engineering applications. Similarly, some of the novel heat exchanger configurations and other heat transfer equipment employ non-circular tubes for the processing of fibrous suspensions, paper making, thermal processing of foodstuffs (carrot/potato chips, for instance) etc. [6]. Further applications are found in the cooling of electronic components and chips of various shapes. In recent years, non-circular tubes are finding increasing applications in specific applications to achieve an improved heat duty and pressure drop performance. Furthermore, such non-circular tubes also offer space economy in terms of the specific heat transfer area.

This work is concerned with the flow of fluids over a semi-circular cylinder. Perhaps one of the most significant features of the bluff body flows is the occurrence of several distinct flow regimes which really denote a balance between various forces prevailing in moving fluids. For instance, for the simplest case of the free flow past an unconfined circular cylinder (long in the transverse direction), the only relevant parameter is the Reynolds number (Re) which is a measure of the relative importance of the inertial and viscous forces. At low Reynolds numbers, the fluid inertia is negligible and therefore, a fluid element