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Short communication

Fuzzy control of original UPOs of unknown discrete chaotic systems

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ABSTRACT

This paper presents a fuzzy algorithm for controlling original unstable periodic orbits of unknown discrete chaotic systems. In the modeling phase, only input–output data pairs provided from the true system are required. The fuzzy model is developed using Gaussian membership functions and consequent functions where the Levenberg–Marquardt computational algorithm is employed for the model parameters calculation. In the controller design phase, the L_2-stability criterion is used, which forms the basis of the main design principle. Simulation results are given to illustrate the effectiveness and control performance of the proposed method.

1. Introduction

Since the research of Ott et al. on chaos control [1], and Pecora and Caroll on chaos synchronization [2], the research subjects of control and synchronization of chaotic dynamical systems have attracted increasing attention from engineering, physics, mathematics and biomedical communities for more than two decades [3,4]. Some existing successful methods and techniques have been reviewed in [5,6].

While most of the researches cited above used conventional control techniques, there has been recent interest in controlling chaos by various artificial intelligence approaches [7,8]. In particular, Sadeghian et al. [9] explained and showed that a fuzzy minimum entropy controller can be employed for stabilizing unstable fixed points of chaotic systems. The proposed fuzzy controller is used to minimize the Shannon entropy of a chaotic dynamics. Controlling of chaos by fuzzy impulsive control was recently developed by Liu and Zhong [10], Zheng and Chen [11], and generalized by Hu et al. [8]. Based on practical aspects, some applications via fuzzy chaos control are available. Especially, Bessa et al. [12] propose that an adaptive fuzzy sliding mode controller can be used for controlling chaotic dynamics with application to a nonlinear pendulum. Controlling of robot manipulators and synchronous motor with the aid of chaos phenomena have been presented in [13] and in [14], respectively.

Apart from that, research over the past two decades shows a rapid development of fuzzy model-based control (FMC) theory which brings up scientist into a new era in controlling nonlinear systems. In fact, the FMC can be applied very well to multivariable dynamical systems [15,16]; this attempt also implies the ability of controlling chaotic systems via fuzzy regulators. Even though it is an interesting research topic to employ chaotic dynamics to develop fuzzy control algorithms. The optimization algorithms proposed in [17,11,8] for stabilizing chaotic systems will generally tend toward unstable fixed points or desired targets. However, unstable periodic orbits embedded in chaotic attractors are fundamental to an understanding of chaotic dynamics. For example, basic ergodic properties such as dimension, Lyapunov exponents, and topological entropy

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can be determined from periodic orbits. Moreover, the detection of such an orbit from experimental data is a test for the presence of determinism. A particularly important application is in the control of chaotic systems where the first essential step is often the determination of periodic orbits. For these reasons, detection of periodic orbits in experimental data has become a central issue. Therefore, to stabilize original unstable periodic orbits (UPOs) embedded in the chaotic system, almost all such UPOs have to be identified with trial and error. That implies the complexity and limit of most of the above approaches.

In this paper, a novel algorithm is presented in order to eliminate the drawbacks of [17] by introducing a fuzzy model-based controller by employing the L_2-stability criteria available to date for stability analysis [18]. Here, the first phase of control is carried out by constructing an attractive domain which allows determination of the original UPOs embedded in the unknown chaotic system. For this purpose, the FMC has the general structure of a combination of Gaussian membership functions and consequent functions. The Gaussian membership functions parameters as well as the consequent functions parameters will be empirically determined from the input–output data pairs by using the Levenberg–Marquardt computational algorithm. Therefore, the Levenberg–Marquardt computational algorithm has the advantage of finding a global solution in the optimization process when it comes to stability analysis. In the second phase, the L_2-stability criterion is used, which forms the basis of the main fuzzy control design principle.

The rest of the paper is organized as follows: in Section 2, concept of the proposed control algorithm is discussed in details. Stability and fuzzy controller scheduling of the closed-loop system are presented in Section 3. In Section 4, three design examples: the Henon system and two discrete systems with simulation results are presented. Finally, concluding remarks are given in Section 5.

2. Fuzzy modeling design

To define fuzzy control for UPO target tracking in an unknown discrete chaotic system, in this section, we describe the fuzzy modeling approach that is characterized by a set of fuzzy rules. The rules of the proposed fuzzy model are expressed in the following forms

$$\begin{aligned}
&R^{(1)}:\ \text{If } x_1 \text{ is } A_1^1 \text{ and } \cdots \text{and } x_n \text{ is } A_n^1,\ \text{then } y_1(x) \text{ is } B^1,\\
&\quad\vdots\\
&R^{(j)}:\ \text{If } x_1 \text{ is } A_1^j \text{ and } \cdots \text{ and } x_n \text{ is } A_n^j,\ \text{then } y_j(x) \text{ is } B^j,\\
&\quad\vdots\\
&R^{(m)}:\ \text{If } x_1 \text{ is } A_1^m \text{ and } \cdots \text{ and } x_n \text{ is } A_n^m,\ \text{then } y_m(x) \text{ is } B^m,
\end{aligned} \tag{1}$$

where $\{x_1,x_2,\ldots,x_n\}$ and $\{y_1,y_2,\ldots,y_m\}$ represent the linguistic variables associated with the input and output of the fuzzy logic system, A_i^j and B^j are labels of the fuzzy sets, index n denotes the number of inputs and m denotes the number of rules.

The consequent functions $y_j(x)$ are defined as follow

$$y_j(x) = B^j = a_{j0} + a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n, \tag{2}$$

with $j = 1,2,\ldots,m$ and a_{ji} ($i = 1,2,\ldots,n$) represent the parameters of the consequent functions $y_j(x)$.

The fuzzy logic system output is given by the following center-average defuzzifier

$$y = \frac{\sum_{j=1}^{m} y_j(x)\mu_j(x)}{\sum_{j=1}^{m}\mu_j(x)}, \tag{3}$$

where the membership functions $\mu_j(x)$ are chosen to be Gaussian. Their centers and relative widths are designed by parameters c_{ji} and σ_{ji}, respectively. Thus, Eq. (3) becomes

$$y = f(x) = \frac{\sum_{j=1}^{m} y_j(x)\mu_j(x)}{\sum_{j=1}^{m}\mu_j(x)} = \frac{\sum_{j=1}^{m}(a_{j0} + a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n)\left(\prod_{i=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_i - c_{ji}}{\sigma_{ji}}\right)^2\right)\right)}{\sum_{j=1}^{m}\left(\prod_{i=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_i - c_{ji}}{\sigma_{ji}}\right)^2\right)\right)}. \tag{4}$$

The fuzzy model can be rewritten by

$$y = f(x) = \theta^T\varphi(x), \tag{5}$$

with θ represents a parameter vector of the following form

$$\theta = [a_{10}, a_{20}, \ldots, a_{m0}, a_{11}, a_{21}, \ldots, a_{m1}, \ldots, a_{1n}, a_{2n}, \ldots, a_{mn}]^T \tag{6}$$

and $\varphi(x) = (\varphi_1(x), \varphi_2(x), \ldots, \varphi_m(x))^T$ is a regressive vector defined by

$$\varphi_j(x) = \frac{\prod_{i=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_i - c_{ji}}{\sigma_{ji}}\right)^2\right)}{\sum_{j=1}^{m}\left(\prod_{i=1}^{n}\exp\left(-\frac{1}{2}\left(\frac{x_i - c_{ji}}{\sigma_{ji}}\right)^2\right)\right)}. \tag{7}$$