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Fundamental frequency of isotropic and orthotropic rectangular plates with linearly varying thickness by discrete singular convolution method

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ABSTRACT

The subject of this article is solving free vibration problems of isotropic and orthotropic rectangular plates with linearly varying thickness along one direction. For the numerical solution to evaluate the frequencies of plates, the method of discrete singular convolution (DSC) is adopted. Frequency parameters are obtained for different types of boundary conditions, taper and aspect ratios. The effect of the mode number is also analyzed. The results obtained by the present numerical method show an excellent agreement with available published results.

1. Introduction

Tapered plates have many applications in civil, aerospace, and mechanical engineering. They are used in the fields such as turbine disk, wings of aircraft, and reinforced bosses. Vibrations of orthotropic parallelogramic plates with variable thickness are given by Dokanish and Kumar [1]. Sakata [2] presented an approximate formula for free vibration of orthotropic rectangular plates with varying thickness. Bert and Malik [3] applied differential quadrature approach for vibration analysis of tapered plates. A comparative study is presented by Kukreti et al. [4] for free vibration analysis of tapered plates. Akiyama and Kuroda [5] obtained fundamental frequency of tapered plates. Free vibration and buckling analysis of rectangular plates with variable thickness are investigated by Ng and Araar [6] using Galerkin method. Chen [7] proposed a modified energy method for bending and free vibration analysis of tapered plates. Appl and Byers [8] calculated the fundamental frequency of linearly varying thickness rectangular plates. Pulmano and Gupta [9] presented a finite strip method for free vibration analysis of tapered plates. Cheung and Zhou [10] applied Rayleigh–Ritz method for vibration analysis of tapered rectangular plates using a new set of beam functions. Effects of boundary constraints and thickness variations on the vibratory response of rectangular plates are investigated by Lim and Liew [11]. Some numerical solution and literature survey for vibration of rectangular plates with abrupt thickness is given by Liew et al. [12–14]. This paper deals with the application of the DSC method for the free vibration analysis of isotropic and orthotropic plates with tapered thickness. To the author' knowledge, by the presented study it is the first time the DSC method has been successfully applied to tapered plate problems for the analysis of free vibration.

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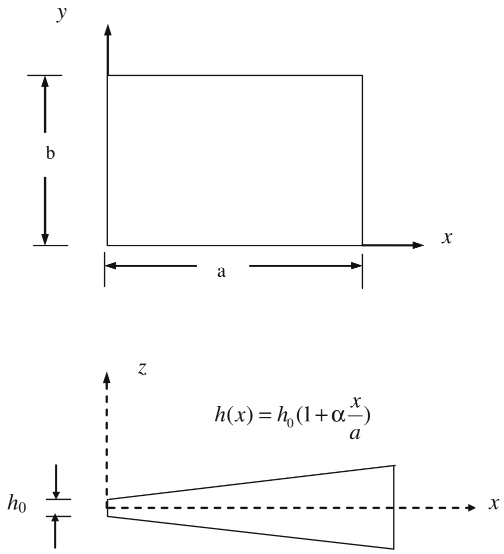

Fig. 1. A typical tapered rectangular plate.

2. Basic formulations

The non-dimensional governing equation for vibration of an orthotropic rectangular plate (Fig. 1) with varying thickness in the x-direction can be given as [3]:

$$H^2\bar{D}_x\frac{\partial^4 W}{\partial X^4}+2\lambda^2H^2(\bar{D}_{xy}+2\bar{G}_{xy})\frac{\partial^4 W}{\partial X^2\partial Y^2}+\lambda^4H^2\bar{D}_y\frac{\partial^4 W}{\partial Y^4}+2H'\bar{D}_x\frac{\partial^3 W}{\partial X^3}+2\lambda^2H'(\bar{D}_{xy}+2\bar{G}_{xy})\frac{\partial^3 W}{\partial X\partial Y^2}$$
$$+H''\left(\bar{D}_x\frac{\partial^2 W}{\partial X^2}+\lambda^2\bar{D}_{xy}+2\bar{G}_{xy}\frac{\partial^2 W}{\partial Y^2}\right)=\Omega^2W, \tag{1}$$

Non-dimensional quantities are given as below:

$$X=x/a, Y=y/b, \lambda=a/b, H=H(x)=h/h_0, H'=\frac{1}{H}\frac{d}{dx}(H^3), H''=\frac{1}{H}\frac{d^2}{dx^2}(H^3)$$
$$\Omega=\omega a^2\sqrt{\rho h_0/D_0},\quad D_0=E_Lh_0^3/12(1-\upsilon_{LT}\upsilon_{TL}),\quad \bar{D}_{xy}=\upsilon_{yx}\bar{D}_x=\upsilon_{xy}\bar{D}_y,$$
$$\bar{G}_{xy}=G_{xy}h_0^3/12D_0,\quad D_x=E_xh_0^3/12(1-\upsilon_{xy}\upsilon_{yx})D_0,\quad D_y=E_yh_0^3/12(1-\upsilon_{xy}\upsilon_{yx})D_0, \tag{2}$$

where a and b are the side lengths of plate along the x- and y-axes of the plate, respectively, W is the deflection, λ is the aspect ratio, h is the thickness of the plate, h_0 is the reference value of plate thickness, Ω is dimensionless frequency, υ_{xy} and υ_{yx} are the Poisson's ratios, E_x and E_y are the in-plane elasticity modulus, G_{xy} is in-plane shear modulus, D_0 is reference value of flexural stiffness, D_x and D_y are the flexural stiffnesses of the plate, D_{xy} is the twisting stiffness, L and T coincident with either of the coordinate axes, are the longitudinal and transverse directions of the given orthotropic material. For example, in the T/L orientation, the transverse axis is coincident with the x-axis.

3. Discrete singular convolution (DSC)

The method of discrete singular convolutions (DSC) was originally introduced by Wei [15] in 1999 as a simple and highly efficient numerical technique. Since then, applications of DSC method to various science and engineering problems have been investigated and their successes have demonstrated the potential of this method as an attractive numerical analysis technique [16–21]. As stated by Wei [22–24] singular convolutions (SC) are a special class of mathematical transformations, which appear in many science and engineering problems, such as the Hilbert, Abel, and Radon transforms.

Wei and his co-workers applied the DSC algorithm for numerical solution of some mechanics problems [25,26]. Zhao et al. [27,28] give the high-frequency vibration analysis of plates using the DSC algorithm. Zhao and Wei [29] adopted the DSC in the vibration analysis of rectangular plates with non-uniform boundary conditions. Numerical solution of free vibration problems of rotating and laminated conical shells and plates on elastic foundation has been given by Civalek [30–33]. These studies indicate that the DSC algorithm works very well for vibration analysis of plates, especially for high-frequency