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Fuzzy weighted equilibrium multi-job assignment problem and genetic algorithm

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ABSTRACT

In this paper, the equilibrium optimization problem is proposed and the assignment problem is extended to the equilibrium multi-job assignment problem, equilibrium multi-job quadratic assignment problem and the minimum cost and equilibrium multi-job assignment problem. Furthermore, the mathematical models of the equilibrium multi-job assignment problem and the equilibrium multi-job quadratic assignment problem with fuzzy parameters are formulated. Finally, a genetic algorithm is designed for solving the proposed programming models and some numerical examples are given to verify the efficiency of the designed algorithm.

1. Introduction

The assignment problem (AP), which has been widely applied in both manufacturing and service systems, is to find the total costs optimal jobs assignment schedule where n jobs are allocated to n workers (or machines), and each worker receives exactly just one job. The AP is a special type of linear 0–1 programming. More generally, there are m jobs and n workers, $m < n$. Each job must be assigned to one and only one worker. Furthermore, the related cost c_{ij} is given with respect to the allocating that the job i is allocated to the worker j. The problem is to find such an assignment that the total costs is minimum. Since the AP is proposed in the literature, there are various extensions.

The first extension is that the costs of the AP are characterized by the random variables and fuzzy variables [1,2]. The quadratic assignment problem (QAP) [3,4,1,5–7,2,8–11] is another important extension of AP where the interactive cost is associated to each edges pair. Furthermore, in some problems each job or worker are associated with a cost when the job is not assigned to any worker or the worker do not receive any job. In such a situation, the AP with penalty (APP) is proposed [12,13]. The equilibrium problem [14] is also an important problem in real-life and the equilibrium matching problem and equilibrium tree problem [14] are also proposed. In this paper, the AP is firstly extended to three types equilibrium models, say the equilibrium multi-job assignment problem, the equilibrium multi-job quadratic assignment problem and the minimum cost and equilibrium multi-job assignment problem, respectively. Then the models of the equilibrium multi-job assignment problem and the equilibrium multi-job quadratic assignment problem with fuzzy parameters are proposed, respectively.

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The multi-job assignment problem is to find a jobs assignment schedule where n jobs are allocated to m workers (or machines), $m < n$ and each worker receives at least one job (multi-job) such that the total cost is minimum. If the problem is to find such a schedule that the difference between the maximum total cost and the minimum total cost with respect to single worker is minimum, then the equilibrium multi-job assignment problem is proposed. In some situations, furthermore, we need to consider the problem to find the multi-job assignment in the optimal equilibrium multi-job assignments such that the costs is minimum. Therefore, the minimum costs and equilibrium multi-job assignment problem is also proposed in this paper.

Since the fuzzy set theory was initialized by Zadeh [15], it has been well developed by many researchers. Recently, Liu [16] developed the credibility theory including the credibility measure, pessimistic value and expected value as fuzzy ranking methods. Based on the credibility theory, the equilibrium multi-job assignment problem with fuzzy parameters are therefore formulated as expected value model, chance-constrained programming, respectively.

This paper is organized as follows. Firstly, the concepts of the equilibrium multi-job assignment problem, the equilibrium multi-job quadratic assignment problem and the minimum costs and equilibrium multi-job assignment problem are initialized and then the mathematical models of the proposed problems are formulated in Section 2. Furthermore, the equilibrium multi-job assignment problem with fuzzy parameters are formulated in Section 3. Finally, a genetic algorithm is designed in Section 4 and some numerical example are given in Section 5.

2. Some notations and the mathematical models

Firstly, let us recall some preliminary notations. Throughout this paper, all the graphs are directed and simple complete bipartite graph which is usually denoted as $G = (V_1,V_2)$, where the vertices set V is defined as $V = V_1 \cup V_2$ and the edges set E is defined as $E = \{(u,v)|u \in V_1,\ v \in V_2\}$. For convenience, we denote $V = \{v_1,v_2,\ldots,v_{|V|}\}E = \{e_1,e_2,\ldots,e_{|E|}\}$. We also employ $E(G)$ and $V(G)$ to denote the edges set and the vertices set of the graph G, respectively. For a subset S of E or V, $G[S]$ denotes the induced subgraph of S. For a vertex $v \in V$, we quote $N_e[v]$ to denote the incident edges set of vertex v.

If the m workers are denoted as set $V_1 = \{1,2,\ldots,m\}$ and n jobs as set $V_2 = \{1,2,\ldots,n\}$. Then the multi-job assignment correspond to a subset of the edges set, which exactly induce a minimum edge cover set of the complete bipartite graph $G = (V_1, V_2)$, of the complete bipartite graph $G = (V_1,V_2)$. A edge cover set C is the set consists of edges such that any vertex $v \in V(G)$ incident to at least edges in C. The minimum edge cover set is the edge cover set with minimum edges in it. An edge $(s,t) \in E(G)$, $s \in V_1, t \in V_2$ implies that the job t an be assigned to worker s. For an edge $(s,t) \in E(G)$, an index or ordering number $(s-1)n + t$ is assigned to edge (s,t). So an edge is also denoted by its index in some places in the following process.

Let $\xi_{ij}, i \in V_1 j \in V_2$ be the direct costs of the edge (i,j) and $\eta_{ij,st}$, $i,s \in V_1 j,t \in V_2$ the interactive costs of the edges (i,j) and (s,t), where $\eta_{ij,st} = 0$ when $(i,j) = (s,t)$ and $\eta_{ij,st} = \eta_{st,ij}$. Let

$$x_{ij} = \begin{cases} 1, & \text{if job } j \text{ is assigned to worker } i, \\ 0, & \text{otherwise.} \end{cases}$$

The vector consists of x_i is denoted as $\boldsymbol{x}$. Obviously, a job assignment is completely characterized by the vector $\boldsymbol{x}$. Then the mathematical model of the standard multi-job assignment problem can be formulated as follows

$$\begin{cases} \min \sum_{i=1}^{m} \sum_{j=1}^{n} \xi_{ij} x_{ij} \\ \text{s.t.} \sum_{j=1}^{n} x_{ij} \geqslant 1, \qquad i = 1,2,\ldots,m, \\ \quad \sum_{i=1}^{m} x_{ij} = 1, \qquad j = 1,2,\ldots,n, \\ \quad x_{ij} = 0 \text{ or } 1. \end{cases} \tag{1}$$

For convenience, four functions are defined as follows:

$$\begin{aligned} f_{\max}(\boldsymbol{x}) &= \max\left\{ \sum_{j=1}^{n} \sum_{j=1}^{n} \xi_{ij} x_{ij} | i \in V_1 \right\}, \\ f_{\min}(\boldsymbol{x}) &= \min\left\{ \sum_{j=1}^{n} \xi_{ij} x_{ij} | i \in V_1 \right\}, \\ g_{\max}(\boldsymbol{x}) &= \max\left\{ \sum_{j=1}^{n} \xi_{ij} x_{ij} + \frac{1}{2} \sum_{j=1}^{m} \sum_{k=1}^{m} \eta_{ij,ik} x_{ij} x_{ik} | i \in V_1 \right\}, \\ g_{\min}(\boldsymbol{x}) &= \min\left\{ \sum_{j=1}^{n} \xi_{ij} x_{ij} + \frac{1}{2} \sum_{j=1}^{m} \sum_{k=1}^{m} \eta_{ij,ik} x_{ij} x_{ik} | i \in V_1 \right\}. \end{aligned} \tag{2}$$