Applied Mathematical Modelling 36 (2012) 2900–2910

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Approach to group decision making based on determining the weights of experts by using projection method

Zhongliang Yue

College of Science, Guangdong Ocean University, Zhanjiang 524088, China

A R T I C L E I N F O

Article history:
Received 27 September 2010
Received in revised form 9 September 2011
Accepted 21 September 2011
Available online 29 September 2011

Keywords:
Multiple attribute group decision making
Projection method
Weight of expert
Ideal decision
Ideal solution

A B S T R A C T

The aim of this paper is to present a new approach for determining weights of experts in the group decision making problems. Group decision making has become a very active research field over the last decade. Especially, the investigation to determine weights of experts for group decision making has attracted great interests from researchers recently and some approaches have been developed. In this paper, the weights of experts are determined in the group decision environment via projection method. First of all, the average decision of all individual decisions is defined as the ideal decision. After that, the weight of expert is determined by the projection of individual decision on the ideal decision. By using the weights of experts, all individual decisions are aggregate into a collective decision. Then an ideal solution of alternatives of the collective decision, expressed by a vector, is determined. Further, the preference order of alternatives are ranked in accordance with the projections of alternatives on the ideal solution. Comparisons with an extended TOPSIS method are also made. Finally, an example is provided to illustrate the developed approach.

1. Introduction

Decision making problem as one of the most important problems in all sciences is the process of finding the best option from all of the feasible alternatives. In many cases, the decision maker needs to take a decision based on multiple attributes to select an alternative from those feasible ones. Multiple attributes decision making (MADM) is an important part of modern decision science, which contains multiple decision attributes and multiple decision alternatives. The aim is to help the decision maker take all important objective and subjective criteria/attributes of the problem into consideration using a more explicit, rational and efficient decision process [1,2]. MADM has been extensively applied to various areas such as society, economics, military, management, etc., and has been receiving more and more attention over the last decades [3,4].

The increasing complexity of the engineering and management environment leads to benefit from a group of experts or decision makers to investigate all relevant aspects of decision making problems [5]. In the recent decade, some studies focused on MADM problems to provide reliable results and take into account the analysis of the experts instead of the analysis of a single expert. This makes that the multiple attributes group decision making (MAGDM) is attracting more and more attention in management, and has received a great deal of attention from researchers [6–12].

The MAGDM problems have three common characteristics: alternatives, multiple attributes with incommensurable units and multiple experts, in which the weights of experts play a very important role, how to determine the weights of experts will be an interesting and important research topic. At present, many methods have been proposed to determine the weights of experts. French [13] proposed a method to determine the relative importance of the group's members by using the

E-mail address: zhongliangyue@gmail.com

0307-904X/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2011.09.068

influence relations, which may exist between the members. Theil [14] proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood [15] and Keeney [16] suggested the use of the interpersonal comparison to determine the scales constant values in an additive and weighted social choice function. Bodily [17] and Mirkin and Fishburn [18] proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Brock [19] used a Nash bargaining based approach to estimate the weights of group members intrinsically. Ramanathan and Ganesh [20] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Martel and Ben Khélifa [21] proposed a method to determine the relative importance of group's members by using individual outranking indexes. Van den Honert [22] used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by the other group members. Jabeur and Martel [23] proposed a procedure which exploits the idea of Zeleny [24] to determine the relative importance coefficient of each member. Chen and Fan [25] proposed a factor score method for obtaining a ranking of the assessment levels of experts in group-decision analysis. By using the deviation measures between additive linguistic preference relations, Xu [26] gave some straightforward formulas to determine the weights of experts. Chen and Fan [27] studied a method for the ranking of experts according to their levels in group decision. Yue [28,29] presented an approach for group decision making based on determining weights of DMs using TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) [30]. Recently, Yue [31] developed a new approach for measuring the decision makers' weights in group decision making setting based on distance measure, in which the decision information is expressed in interval-valued intuitionistic fuzzy numbers.

The existing approaches dealing with the weights of experts can be divided into two categories: the subjective preference information of expert is represented by Saaty's multiplicative preference relation [12] and others, including subjective and objective preference information taken the form of real numbers [29], interval numbers [28], language [26,27], and other [25,31].

Most of the existing approaches are to take the form of Saaty's multiplicative preference relation. The disadvantages of these approaches are that subjectivity of experts is too strong and the procedure dealing with the weights of experts is very complicated. To resolve these problems, by using the TOPSIS, Yue [28,29,31] developed some methods for determining weights of experts.

In this study, we propose a straightforward and practical method to deriving the weights of experts and ranking the preference order of alternatives based on projection method [4,32–35]. Projection method is used twice to the developed approach in this paper, which is first used to determine the weights of experts, and second used to rank the preference order of alternatives.

The reminder of this paper is organized as follows: in the next section, briefly introduces the projection method. In Section 3, we present an algorithm for MAGDM based on determining the weights of experts using projection method. In Section 4, we make some comparisons between the presented method and an extended TOPSIS method. In Section 5, we illustrate our proposed algorithmic method with an example. The final section concludes.

2. Projection method

Definition 1 [4]. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be a vector, then

$$|\alpha| = \sqrt{\sum_{j=1}^{n} \alpha_j^2} \tag{1}$$

is called the module of vector α.

Definition 2 [4]. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ be two vectors, then

$$\alpha\beta = \sum_{j=1}^{n} \alpha_j \beta_j \tag{2}$$

is called the inner product between α and β.

Through a combination of Eqs. (1) and (2), we have the concept of projection between two vectors as follows:

Definition 3 ([4,32,33]). Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ be two vectors, then

$$Prj_{\beta}(\alpha) = |\alpha| cos(\alpha, \beta) = |\alpha| \frac{\alpha\beta}{|\alpha||\beta|} = \frac{\alpha\beta}{|\beta|} \tag{3}$$

is called the projection of the vector α on the β.

The projection can be illustrated in Fig. 1.

In general, the bigger the value of $Prj_{\beta}(\alpha)$, the more the degree of the vector α approaching to the vector β.