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Analysis of algebraic multigrid parameters for two-dimensional steady-state heat diffusion equations

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ABSTRACT

In this work, it is provided a comparison for the algebraic multigrid (AMG) and the geometric multigrid (GMG) parameters, for Laplace and Poisson two-dimensional equations in square and triangular grids. The analyzed parameters are the number of: inner iterations in the solver, grids and unknowns. For the AMG, the effects of the grid reduction factor and the strong dependence factor in the coarse grid on the necessary CPU time are studied. For square grids the finite difference method is used, and for the triangular grids, the finite volume one. The results are obtained with the use of an adapted AMG1R6 code of Ruge and Stüben. For the AMG the following components are used: standard coarsening, standard interpolation, correction scheme (CS), lexicographic Gauss–Seidel and *V*-cycle. Comparative studies among the CPU time of the GMG, AMG and singlegrid are made. It was verified that: (1) the optimum inner iterations is independent of the multigrid, however it is dependent on the grid; (2) the optimum number of grids is the maximum number; (3) AMG was shown to be sensitive to both the variation of the grid reduction factor and the strong dependence factor in the coarse grid; (4) in square grids, the GMG CPU time is 20% of the AMG one.

1. Introduction

The multigrid method [1,2] belongs to a group of iterative solvers, and it is one of the most efficient and widespread methods to solve large systems of linear equations [3]. Its efficiency is based on the fact that the multigrid method presents a potential to solve $N \times N$ linear systems with only $O(N)$ computational effort [4]. Two different approaches can be accomplished employing the multigrid method according to the kind of data and information employed and also how the operators deal with them: the geometric multigrid (GMG) and the algebraic multigrid (AMG). The main difference between AMG and GMG is related to the manner of constructing the coarser grids [5]: the AMG method requires no knowledge of the geometry of the problem [6]. The GMG method [1,2] employ fixed grid hierarchies and, therefore, an efficient interplay between smoothing and coarse-grid correction has to be ensured by selecting appropriate smoothing processes [1]. On the other hand, the AMG method [5–8] fixes the smoother to some simple relaxation scheme and enforces an efficient interplay with coarse-grid correction by choosing the suitable coarser levels and the interpolation [1]. The grid hierarchies in the AMG is

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generated in the setup phase (which is an initial or start-up phase), by considering the coefficient matrix, as well as the build interpolation, restriction and coarse-grid operators [6,9].

The application of the AMG method includes problems in which the use of the GMG method is difficult or even impracticable [7], such as: unstructured grids, large matrix equations which are not at all derived from continuous problems, extreme anisotropic equations and so on. A remarkable use of the AMG method takes place when there is none information about the problem geometry [9]. Table 1, adapted from Chang et al. [10], provides a comparison between the AMG and the GMG methods.

Both strategies, GMG and AMG, present similar steps. The first one is the generation of the coarser grids (levels), which is followed by the transfer of information among different grids (restriction and prolongation operators). After that comes the solution of the linear system in each grid using an iterative smoother algorithm (solver), and the choice of a multigrid cycle (*F*-cycle, *V*-cycle, *W*-cycle, among other ones) [1,2]. The decision about which iterative method to use as smoother (solver), how to coarsen the problem, or even how to transfer information between the grids, often involve considerable algorithmic research [4].

According to Trottenberg et al. [1], a single modification in the algorithm might result in a significant reduction of the CPU time requirements. The efficiency of the multigrid methods is also related to the adaptations of the multigrid components, which should be made properly according to the underlying physical problem and the variational formulation [11]. Unfortunately, the works available in the literature do not present deep studies about the components of the AMG algorithm and their optimization. More precisely, in such works, new coarsening algorithms and/or new interpolation operators are introduced, like the work of Xiao et al. [12], or to papers in which the AMG performance is compared to the GMG one [11,13]. In the latter case, such comparisons are limited to the CPU time, the number of cycles and the study of the multigrid efficiency provided by the speedup value.

The CPU time and its growth, according to the number of unknowns, were studied by Watanabe et al. [13], for both AMG and GMG. Both multigrid methods were also studied by Langer and Pusch [11], where comparisons for the number of cycles spent by the AMG and by the GMG, as well as the time requirements for the auxiliary grids generation were presented. The number of cycles was also revised by Wu and Elman [14], by using as stop criterion a given tolerance value; it was seen that the GMG convergence was slower than the AMG for convection–diffusion problems. Campos et al. [15] made a comparison between the performances of the AMG and the GMG, both with parallelized and preconditioned algorithms which are suitable for a non-linear system of differential equations. Additionally, simulations were performed varying the number of grids, for both the AMG and the GMG, and the grid reduction factor for the AMG, executed by 1, 2, 4 or 6 processors; the performance of the AMG algorithm was better than the one obtained by the GMG for both CPU time and memory.

Systematic studies about the multigrid parameters where found only for the GMG method. Gaspar et al. [16,17] presented theoretical and numerical results for the GMG with triangular grids, by applying distinct multigrid cycles, different numbers of inner iterations and proposing a new smoother (solver). On the other hand, Oliveira et al. [18] computed optimum values for the inner iterations and the levels used in heat diffusion problems with structured square grids.

Motivated by all these previous works, the purpose of this paper is to study the components and the parameters associated to the AMG method, by applying the numerical AMG1R6 code implemented by Ruge and Stüben [7], in order to find the optimum parameters for the AMG. The meaning of optimum values for each AMG parameter, considered here, is associated to the minimization of the CPU time, i.e., the values for each AMG parameter which corresponds to the smallest CPU time in each simulation, keeping fixed the values for the other parameters.

Two-dimensional Laplace and Poisson type equations are employed, using both square structured (both equations) and triangular grids (Laplace equation), in order to investigate the influence of the number of inner iterations (ν) and the number of levels (L) for both AMG and GMG methods, providing comparisons on the performances of such methods. Moreover, in the case of AMG, two parameters (which influence the coarser grids generation) are also investigated: the grid reduction factor (θ) and the strong dependence factor in the coarse grid (ε).

Although most works present the values used for each AMG parameter, or at least, some of them [1,2,6,7,14,19,20], none of these papers explain clearly why such values are employed; in most cases, it is only said that the used value is a universal practice, without giving more details. Based on these facts, this work tries to fill this lack of information providing the optimized values for the AMG parameters, compared to the standard ones.

As usual, the standard AMG algorithm adopts the following values for its parameters: $\nu = 1$ [1,2,6,7,19,20], $L = L_{maximum}$ [14,19,20], $\theta = 0.25$ [1,7,10,19,20] and $\varepsilon = 0.35$ [7,19]. In some works available in the literature [1,2,6], the authors employ values other than the ones listed here. However, they do not clearly explain the reasons which take them to use such values.

Table 1
Comparison between the GMG and the AMG methods (adapted from Chang et al. [10]).

Features	GMG	AMG
Solved problem	Continuous problem	Linear systems of algebraic equations
Used information	Geometrical structure of the problem	Only entries of the matrix
Program	Necessity of composing a program for each problem	Only one program for different problems
Efficiency	Very good	Good