Applied Mathematical Modelling 36 (2012) 3007–3028

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A new risk criterion in fuzzy environment and its application

Yanju Chen, Yankui Liu *, Xiaoli Wu

College of Mathematics & Computer Science, Hebei University, Baoding 071002, Hebei, China

ARTICLE INFO

Article history:
Received 20 August 2010
Received in revised form 27 September 2011
Accepted 29 September 2011
Available online 10 October 2011

Keywords:
Risk measure
Absolute semi-deviation
Fuzzy portfolio optimization
Fractional programming
Pseudo-convexity
Domain decomposition method

ABSTRACT

Since the observed values of security returns in real-world problems are sometimes imprecise or vague, an increasing effort in research is devoted to study the properties of risk measures in fuzzy portfolio optimization problems. In this paper, a new risk measure is suggested to gauge the risk resulted from fuzzy uncertainty. For this purpose, the absolute deviation and absolute semi-deviation are first defined for fuzzy variable by nonlinear fuzzy integrals. To compute effectively the absolute semi-deviations of single fuzzy variable as well as its functions, this paper discusses the methods of computing the absolute semi-deviation by classical Lebesgue–Stieltjes (L–S) integral. After that, several useful absolute deviation and absolute semi-deviation formulas are established for common triangular, trapezoidal and normal fuzzy variables. Applying the absolute semi-deviation as a new risk measure in portfolio optimization, three classes of fuzzy portfolio optimization models are developed by combining the absolute semi-deviation with expected value operator and credibility measure. Based on the analytical representation of absolute semi-deviations, the established fuzzy portfolio selection models can be turned into their equivalent piecewise linear or fractional programming problems. Since the absolute semi-deviation is a piecewise fractional function and pseudo-convex on the feasible subregions of deterministic programming models, we take advantage of the structural characteristics to design a domain decomposition method to separate a deterministic programming problem into three convex subproblems, which can be solved by conventional solution methods or general-purpose software. Finally, some numerical experiments are performed to demonstrate the new modeling idea and the effectiveness of the solution method.

1. Introduction

In financial theory, especially in portfolio optimization, much research has been done to identify the properties of risk measures. One aim of the research was to develop an axiomatically founded risk theory in finance [1], and another goal was to provide guidelines for practitioners to select an appropriate risk measure in their daily work. The seminal Markowitz's mean–variance method employed the variance as the risk measure [2], which has been widely accepted as a practical tool for portfolio optimization. The use of the semivariance rather than variance as the risk measure was also proposed by Markowitz [3]. Since then, several other risk measures have been documented in portfolio literature. For example, Konno and Yamazaki [4] measured investment risk by the absolute deviation and developed mean-absolute deviation models; Jorion [5] studied Value-at-Risk (VaR) as a risk measure and applied mean-VaR model in finance industry, and Rockafellar and Uryaser [6] reduced investment risk by minimizing conditional Value-at-Risk (CVaR) and established mean-CVaR model. In addition to various risk measures, stochastic dominance [7] is another important approach to modeling the choice among uncertain

* Corresponding author. Tel./fax: +86 312 5066629.
E-mail addresses: chen-yanju@163.com (Y. Chen), yliu@hbu.edu.cn (Y. Liu), wxl861@126.com (X. Wu).

0307-904X/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2011.09.081

outcomes. Ogryczak and Ruszczyński [8] studied the consistency of stochastic dominance with some risk measures such as the standard semi-deviation and absolute semi-deviation of random variable.

The conventional portfolio methods require the security returns are random variables, and probability theory is the main research tool. However, the observed values of security returns in real-world problems are sometimes imprecise or vague. Imprecise evaluations may result from unquantifiable, incomplete and non obtainable information. Since the seminal works of Zadeh [9,10], fuzzy set and possibility theory have become prominent tools for handling imprecision or vagueness aiming at tractability, robustness and low cost solutions for real-world problems [11–13]. In portfolio analysis, a great deal of achievements have been made based on fuzzy set and possibility theory. For example, Watada [14] discussed portfolio selection by using fuzzy decision theory; Tanaka and Guo [15] used possibility distributions to model uncertainty in the returns and identified upper and lower possibility distributions from the given possibility degrees to security data; Inuiguchi and Ramík [16] exemplified the advantages and disadvantages of fuzzy mathematical programming approaches in the setting of an optimal portfolio selection problem; Inuiguchi and Tanino [17] introduced a possibilistic programming approach to the portfolio selection problem under the minimax regret criterion; Arenas-Parra et al. [18] discussed the optimal portfolio for a private investor by taking into account three criteria: return, risk and liquidity; León et al. [19] dealt with fuzzy optimization schemes for managing a portfolio in the framework of risk-return trade-off; Carlsson et al. [20] introduced a possibilistic approach for selecting portfolios with the highest utility value under the assumption that the returns of assets are trapezoidal fuzzy numbers; Lin et al. [21] proposed a systematic approach by incorporating fuzzy set theory in conjunction with portfolio matrices to assist managers in reaching a better understanding of the overall competitiveness of their business portfolios; Fang et al. [22] proposed a portfolio rebalancing model with transaction costs based on fuzzy decision theory; Dastkhan et al. [23] studied a linguistic-based portfolio selection model by weighted max–min operator and designed a hybrid genetic algorithm to solve it, and Zhang et al. [24] dealt with the portfolio adjusting problem for an existing portfolio under the assumption that the returns of risky assets are fuzzy numbers and there exist transaction costs in portfolio adjusting process. Moreover, on the basis of credibility measure [25,26], some scholars contributed several risk measures to help selecting optimal portfolios. Chen et al. [27] constructed mean–variance models with security returns characterized by fuzzy variables with known possibility distributions; Qin et al. [28] discussed the Kapur cross-entropy minimization model for portfolio selection problem under fuzzy environment, which minimizes the divergence of the fuzzy investment return from a priori one; Zhang et al. [29] discussed portfolio adjusting problems for an existing portfolio, in which the returns of risky assets are regarded as fuzzy variables and a class of mean–variance adjusting models with transaction costs are proposed based on credibility measure; Huang [30] employed the semivariance to describe asymmetry of fuzzy returns; Li et al. [31] used the skewness of fuzzy returns to characterize the corresponding asymmetry; Wu and Liu [32] developed the mean-spread models for fuzzy portfolio selection problem to avoid the difficulty of computing the variance of fuzzy variable, and Huang [33] proposed two credibility-based minimax mean–variance models, where each security return belongs to a certain class of fuzzy variables but the exact fuzzy variable cannot be given. For recent developments about fuzzy portfolio selection problems, the interested reader may also refer to the review papers [34,35].

Thus far, to the best of our knowledge, there is no research on fuzzy portfolio optimization taking absolute semi-deviation as risk measure. It is known that the absolute deviation is an important risk measure in stochastic portfolio selection [4], and taking the absolute semi-deviation as risk measure, the optimization model for minimizing the corresponding objective function will deliver the same results as its absolute deviation counterpart [36]. In addition, the absolute semi-deviation as a risk measure is consistent with stochastic dominance [8], which is a frequently used method in finance for modeling the choice among uncertain outcomes. Under these considerations, in this paper, we will extend stochastic absolute deviation idea to fuzzy decision system. We first define the absolute semi-deviation of fuzzy variable and discuss its L–S integral calculus, and then apply the absolute semi-deviation risk criterion in fuzzy portfolio selection problem. However, in fuzzy environments, taking the absolute semi-deviation as a risk measure, the optimization model for minimizing the corresponding objective function will obtain different results from its absolute deviation counterpart. From this viewpoint, the absolute semi-deviation method developed in this paper provides an alternative method to gauge downside risk for asymmetric fuzzy returns. In addition, the absolute semi-deviation risk measure has the following advantages over some existing fuzzy methods that gauge the risk brought out by asymmetric fuzzy returns. First, under mild assumptions, the absolute semi-deviation of single fuzzy variable as well as its functions can be expressed as classical L–S integrals, which facilitate us to compute the absolute semi-deviation of fuzzy variable by using L–S integral calculus [37]. Second, taking the absolute semi-deviation as a risk measure, for common fuzzy return rates, the mean-absolute semi-deviation portfolio selection problems can be turned into their equivalent piecewise linear or fractional programming ones, which can be solved by combining domain decomposition method and general-purpose software. Therefore, from the algorithmic viewpoint, the absolute semi-deviation method has some advantages over other risk measures that gauge asymmetric fuzzy returns such as semivariance.

The rest of this paper is organized as follows. In Section 2, we first recall some basic concepts about fuzzy variable, and then define absolute deviation and absolute semi-deviations of fuzzy variable. In Section 3, we discuss the computational methods for absolute semi-deviations. Under mild assumption, the absolute semi-deviation of single fuzzy variable as well as its functions can be computed by L–S integral calculus. For common normal, triangular and trapezoidal fuzzy variables, Section 4 derives their absolute deviation and absolute semi-deviations formulas. In Section 5, we first develop three classes of fuzzy portfolio optimization models by combining the absolute semi-deviation with expected value operator and credibility measure, and then discuss their equivalent deterministic programming problems. By using the structural characteristics of absolute semi-deviation analytical expression, Section 5 also designs a domain decomposition method to separate a