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Adaptive level set evolution starting with a constant function

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ABSTRACT

In this paper, we propose a novel level set evolution model in a partial differential equation (PDE) formulation. According to the governing PDE, the evolution of level set function is controlled by two forces, an adaptive driving force and a total variation (TV)-based regularizing force that smoothes the level set function. Due to the adaptive driving force, the evolving level set function can adaptively move up or down in accordance with image information as the evolution proceeds forward in time. As a result, the level set function can be simply initialized to a constant function rather than the widely-used signed distance function or piecewise constant function in existing level set evolution models. Our model completely eliminates the needs of initial contours as well as re-initialization, and so avoids the problems resulted from contours initialization and re-initialization. In addition, the evolution PDE can be solved numerically via a simple explicit finite difference scheme with a significantly larger time step. The proposed model is fast enough for near real-time segmentation applications while still retaining enough accuracy; in general, only a few iterations are needed to obtain segmentation results accurately.

1. Introduction

Image segmentation has been, and still is, an important area in image analysis and computer vision. For a given image, its goal is to separate the image domain into a set of regions which are visually distinct and uniform with respect to certain properties, such as grey level, texture or color.

To perform the image segmentation task, many successful techniques including geometric active contour models using the level set method [1] have been presented. The use of level set method allows us to handle complex geometry and even changing topology, without the need of user-interaction [2]. According to the way representing the surface, applications of the level set method in image segmentation can be typically divided into two classes: the standard level set method [3–9] and the piecewise constant level set method [10–12]. The surface is represented by the zero level set of a Lipschitz function in the standard level set method, while it is represented by discontinuities of a piecewise constant function in the piecewise constant level set framework. In this paper, we confine our discussion to the standard level set method for image segmentation.

In the standard level set method [1,3,4], the level set function can develop shocks, very sharp and/or flat shape during the evolution, which makes further computation highly inaccurate. To avoid these problems, a common numerical scheme is to initialize the level set function to a signed distance function to initial contour before the evolution, and then to periodically re-initialize the level set function to be a signed distance function to the evolving curve during the evolution. Indeed, such initialization and re-initialization are crucial and cannot be avoided in the standard level set method. From the practical viewpoints, however, such initialization and re-initialization are fraught with its own problems, such as the determinations

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of where to define the initial contour and when and how to re-initialize the level set function to a signed distance function [13]. Recently, Li et al. [6,7] proposed a novel level set evolution model without re-initialization. It employed a deviation penalization energy to preserve the level set function close to a signed distance function, thus not only the re-initialization is entirely eliminated, but also the level set function can be initialized to a piecewise constant function. Unfortunately, the initial piecewise constant function also needs to be computed from an initial contour or several initial contours that partition the image domain into different regions; it still has the problems resulted from contours initialization, such as how and where to define the initial contours.

In this paper, we propose a new level set evolution model in a partial differential equation (PDE) formulation. According to the evolution PDE, the level set evolution is controlled by two forces, an adaptive driving force that ensures the level set function to adaptively move up or down according to image information, a TV (total variation)-based regularizing force that smoothes the level set function. Due to the adaptive driving force, the level set function can be simply initialized as a positive constant function rather than the signed distance function [3–5,9] or piecewise constant function [6–8], and thus completely eliminates the need for initial contours. Also, re-initialization is not necessary in our model because the level set function is no longer required to keep as a signed distance function. In addition, a simple explicit finite difference scheme with a significantly larger time step can be used for solving the evolution PDE numerically.

The remainder of this paper is organized as follows. Section 2 reviews an early geometric active contour [3], the level set evolution model without re-initialization [6,7] and their limitations. The idea of TV-based regularization [14] in image restoration is also summarized in this section. The proposed model is introduced in Section 3. Section 4 presents numerical algorithm and validates our model by experiments, followed by the discussions of the parameters in Section 5. This paper is summarized in Section 6.

2. Related works

2.1. An early geometric active contour

An early geometric active contour, introduced by Caselles et al. [3], is based on the theory of curve evolution [15] and the level set method [1].

Let $\phi(x,y,t)$ be a Lipschitz function whose zero level set defines the evolving curve $C(x,y,t)$. In the level set method, the evolution of the curve $\phi(x,y,t)$ along its normal direction with speed F is implicitly defined via the following nonlinear evolution PDE [1]:

$$\frac{\partial \phi}{\partial t} = F|\nabla \phi|, \tag{1}$$

with initial condition $\phi(x,y,0) = \phi_0(x,y)$, where $\phi_0(x,y)$ is the initial level set function corresponding to the initial curve $C(x,y,0)$.

A particular case is the motion derived by mean curvature, where $F = \text{div}\,(\nabla\phi/|\nabla\phi|) = \kappa$ is the curvature of the level curve of ϕ passing though (x,y). The Eq. (1) becomes

$$\frac{\partial \phi}{\partial t} = \kappa|\nabla \phi|. \tag{2}$$

An early geometric active contour model [3] based on Eq. (2) is given by the following evolution equation:

$$\begin{cases} \frac{\partial \phi}{\partial t} = g(|\nabla I)(\kappa + v)|\nabla \phi| \\ \phi(x,y,0) = \phi_0(x,y), \end{cases} \tag{3}$$

where $v \geqslant 0$ is a constant serving as a balloon force, and $g(|\nabla I|) = \frac{1}{1+|\nabla(G_\sigma * I)|^2}$ is an edge-stopping function derived from the image. In Eq. (3), the term $g(|\nabla I|)(\kappa + v)$ determines the overall evolution speed of level sets of $\phi(x,y,t)$ along their normal direction. The use of curvature κ has the effect of smoothing the contour, while the use of v has the effect of shrinking or expanding contour at a constant speed. The speed of contour evolution is coupled with the image data through a multiplicative stopping term $g(|\nabla I|)$.

In implementing this model via the level set method, it is necessary to initialize the level set function to a signed distance function and periodically reshape it during the evolution by the following PDE [16]:

$$\frac{\partial \phi}{\partial t} = sign(\hat{\phi})(1 - |\nabla \phi|), \tag{4}$$

with initial condition $\phi(x,y,0) = \hat{\phi}(x,y)$, where $\hat{\phi}$ is the function to be re-initialized, and $sign(\cdot)$ is the signum function. When the steady state of the initial value problem (4) is reached, ϕ will be a distance function having the same zero level set as $\hat{\phi}$. This is commonly known as the re-initialization procedure in the level set formwork.

The initial signed distance function is defined by an initial contour or several initial contours in the image domain. Naturally, the problem of contour initialization arises; we have to choose suitable initial contours to correctly detect objects of interest in a given image, e.g., the initial contour must encircle all the objects to be detected or several contours must be