Applied Mathematical Modelling 36 (2012) 3312–3319

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Adaptive Q–S synchronization between coupled chaotic systems with stochastic perturbation and delay

Jiakun Zhao *

School of Science, Xi'an Jiaotong University, Xi'an 710049, China

ARTICLE INFO

Article history:
Received 8 June 2011
Received in revised form 25 September 2011
Accepted 16 October 2011
Available online 21 October 2011

Keywords:
Q–S synchronization
Adaptive control
Hyper-chaotic system
Unknown parameters
Invariance principle
Stochastic differential equation

ABSTRACT

This work investigates the adaptive Q–S synchronization of coupled chaotic (or hyper-chaotic) systems with stochastic perturbation, delay and unknown parameters. The sufficient conditions for achieving Q–S synchronization of two stochastic chaotic systems are derived based on the invariance principle of stochastic differential equation. By the adaptive control technique, the control laws and the corresponding parameter update laws are proposed such that the stochastic Q–S synchronization of non-identical chaotic (or hyper-chaotic) systems is to be obtained. Finally, two illustrative numerical simulations are also given to demonstrate the effectiveness of the proposed scheme.

1. Introduction

Synchronization of chaotic dynamical systems has attracted a growing interest with applications in various fields [1,2]. In the context of coupled chaotic elements, many different types of synchronization have been studied in the past two decades. The most important ones are complete or identical synchronization (CS) [3–5], phase synchronization (PS) [6,7], lag synchronization (LS) [8] and generalized synchronization (GS) [9,10]. In recent years, chaotic synchronization has been investigated extensively by many researchers, and various modern control methods have been proposed to synchronize chaotic systems, such as backstepping design [11], linear feedback control [12], nonlinear control [13], adaptive control [14], active control [15], or adaptive-active control [16]. However, most of above schemes are just presented for un-bi-directionally coupled systems, and do not consider the effect of noise in the synchronized process. Since many systems should be described by bi-directionally coupled systems and noise is ubiquitous in both nature and man-made systems, chaos synchronization is unavoidably subject to bi-directionally coupled systems and internal and external stochastic perturbations. Therefore, research of noise's role in synchronization has become a hot field [17–20].

It is also important to note that systems should hold time delay in noise and unknown parameter mismatch. The chaotic systems are inevitably exposed to an environment which may cause their parameters a little different, and it is also difficult to estimate this mismatch of parameters exactly. In [21], an adaptive controller has been designed to synchronize the noise-perturbed two bi-directionally coupled chaotic systems with time delay and unknown parametric mismatch. But to our knowledge, effects of noise in generalized synchronization have not considered adequately. For instance, Q–S synchronization is more general one firstly introduced by Yan [22]. Then, inspired by the works [21,22], we use adaptive control to implement a particular kind of synchronization with noise perturbation where the driving system and the response are

* Tel.: + 86 29 82668741.
E-mail address: zhaojk@mail.xjtu.edu.cn

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doi:10.1016/j.apm.2011.10.029

all perturbed by white noise with delay. Based on the above reason and the so-called LaSalle-type invariance principle for stochastic differential equation proposed by Mao [23], we describe this kind of synchronization as stochastic Q–S synchronization with unknown parameters, which is less restrictive but more extensive.

The rest of this paper is organized as follows. The problem formulation for the adaptive stochastic Q–S synchronization is given in Section 2. Numerical examples are provided to illustrate the effectiveness of the obtained scheme in Section 3. Conclusions and further works are finally drawn in Section 4.

2. Preliminaries and problem formulation

To make this paper self-contained and since the theory of stochastic differential equation is not well-known to the scientific community, we address some basic results in this section which is useful in the following. These results can be found in [23].

Consider the general n-dimensional stochastic differential equation with time delay

$$\dot{x} = f(x, x_\tau, t) + \sigma(x, x_\tau, t)\dot{W}, \tag{2.1}$$

where $x_\tau = x(t-\tau)$. Then, for each $V \in C^{2,1}(R^n \times R_+, R_+)$, define the diffusion operator L along with (2.1) could be express by

$$\mathcal{L}V(x,y,t) = V_t(x,t) + V_x(x,t)f(x,y,t) + \frac{1}{2}trace(\sigma^T(x,y,t)V_{xx}\sigma(x,y,t)), \tag{2.2}$$

where

$$V_t(x,t) = \frac{\partial V(x,t)}{\partial t}, \quad V_x(x,t) = \left(\frac{\partial V(x,t)}{\partial x_1}, \ldots, \frac{\partial V(x,t)}{\partial x_n}\right) \quad \text{and} \quad V_{xx}(x,t) = \left(\frac{\partial^2 V(x,t)}{\partial x_i x_j}\right)_{n\times n}.$$

So the LaSalle-type invariance principle for stochastic differential equation with time delay can be expressed as follows (called **Invariance principle**):

(1) Assume that system (2.1) has a unique solution $x(t,\phi)$ on $t \geqslant 0$ for any initial point $\phi \in C_F^\alpha([-\tau,0],R^n)$. Moreover, both $f(x,y,t)$ and $\sigma(x,y,t)$ are local bounded in (x,y) while uniformly bounded in t.
(2) Assume also that there are functions $V \in C^{2,1}(R^n \times R_+, R_+)$, $\gamma \in L^1(R_+,R_+)$, and ω_1, $\omega_2 \in C(R^n,R_+)$ such that

$$\mathcal{L}V(x,y,t) \leqslant \gamma(t) - \omega_1(x) + \omega_2(y), \quad (x,y,t) \in R^n \times R^n \times t, \tag{2.3}$$

$$\omega_1(x) \geqslant \omega_2(x), \quad x \neq 0, \tag{2.4}$$

$$\lim_{|x|\to+\infty} \inf_{0\leqslant t\leqslant\infty} V(x,t) = \infty. \tag{2.5}$$

Then

$$\lim_{t\to+\infty} x(t,\phi) = 0 \quad \text{a.s.} \tag{2.6}$$

for every $\phi \in C_F^\alpha([-\tau,0],R^n)$.

Consider an m-dimensional chaotic (hyper-chaotic) system described by

$$\dot{x} = f(x) + F(x)\mathrm{P} + \sigma_1(x_\tau, y_\tau)\dot{W}_1, \tag{2.7}$$

where $x = (x_1,x_2\ldots x_m)^T \in R^m$ is the state vector of the system, $f \in C(R^m,R^m)$ including nonlinear terms, $F \in C(R^m,R^{m\times k})$, $\mathrm{P} \in R^k$ is the vector of system parameters and $\sigma_1(x,y) \in C(R^m \times R^n, R^{m\times m_1})$ is continuous nonlinear matrix-valued functions. $\left(\dot{W}_1^T, \dot{W}_2^T\right) = (\eta_1,\eta_2\ldots\eta_{m_1},\eta_{m_1+1}\ldots\eta_{m_1+n_1})$ is an (m_1+n_1)-dimensional white noise in which every two elements is statistically independent, i.e., $E[\eta_1] = 0$, $E[\eta_i(t)\eta_j(t')] = \delta_{ij}\delta(t-t')$ $(i,j = 1,2\ldots m_1+n_1)$. We take Eq. (2.7) as the drive system. The controlled response system is given by

$$\dot{y} = g(y) + G(y)\Theta + \sigma_2(y_\tau, x_\tau)\dot{W}_2 + u, \tag{2.8}$$

where $y = (y_1,y_2\ldots y_n)^T \in R^n$ is the state vector, $g \in C(R^n,R^n)$ including nonlinear terms, $G \in C(R^n,R^{n\times l})$, $\Theta \in R^l$ is the parameter vector and $\sigma_2(y,x) \in C(R^n \times R^m, R^{n\times n_1})$ is continuous nonlinear matrix-valued functions. The purpose of stochastic synchronization is to design a controller $u(u \in R^n)$, which is able to Q–S synchronize the two identical or different chaotic (hyper-chaotic) systems with delay. Let $Q(x) = (Q_1(x),Q_2(x)\ldots Q_h(x))^T$ and $S(y) = (S_1(y),S_2(y)\ldots S_h(y))^T$ be observable variable of the system (2.7) and the system (2.8), respectively. Let the stochastic synchronization error of the two chaotic (hyper-chaotic) systems be

$$e(t) = Qx - Sy, \tag{2.9}$$

where $Q \in R^{h\times m}$, $S \in R^{h\times n}$are two matrixes, $h \leqslant \min\{m,n\}$. Then the dynamical system between the drive system (2.7) and the response system (2.8) can be written as