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Reflection and refraction of a plane thermoelastic wave at a solid–solid interface under perfect boundary condition, in presence of normal initial stress

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ABSTRACT

Using the basic governing equations for isotropic and homogeneous generalized thermo elastic media under initial stress, the reflection and refraction of thermo elastic plane waves at the interface of two dissimilar thermo elastic solid half-spaces has been investigated. The amplitude ratios of various reflected and refracted waves are obtained for an ideal boundary for the incidence of SV-wave. The numerical computations are carried out for a particular model. The effect of initial stress on the amplitude ratios are shown graphically after numerical calculation.

1. Introduction

Initial stresses develop in the medium due to various reasons, such as the difference of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, and gravity variations. The Earth is under high initial stress and therefore, it is of great interest to study the effect of these stresses on the propagation of elastic waves. A lot of systematic studies have been made on the propagation of elastic waves. Biot [1] showed that the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. Based on Biot's theory, Montanaro [2] investigated the isotropic linear thermo elasticity with hydrostatic initial stress. Dey and Addy [3] considered the effect of initial stress on the reflection of waves at the solid half-space. Chattopadhyay et al. [4] studied Reflection of elastic waves under initial stress at a free surface. Sidhu and Singh [5] reviewed the problem of reflection of elastic waves under initial stress at a free surface. Dey et al. [6] considered reflection and refraction of P-waves under initial stresses at an interface. However none of them considered the effect of temperature field along with the initial stress. Addy and Chakraborty [7,8] studied the elastic wave propagation in presence of initial tress and temperature fields both, under the effect of gravity and also in visco-elastic medium. The general equation of reflection and refraction of elastic waves at a plane half-space were formulated by Knott [9]. Gutenberg [10] calculated the square roots of the energy ratio of the reflected and refracted seismic waves while Jeffreys [11] calculated the reflection and refraction coefficients. But none of them considered the presence of initial stress in the solid half-space. Deresiewcz [12] and Beevers and Bree [13] also studied the reflection problems in linear coupled thermo elasticity without considering initial stress in the medium. Duhamel [14] introduced the theory of

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Nomenclature

$U(x, y, t)$	displacement component along x-axis
$V(x, y, t)$	displacement component along y-axis
$T(x, y, t)$	incremental change of temperature
T_0	initial uniform temperature in absolute scale
λ, μ	Lame's constants
α_t	coefficient of linear thermal expansion
γ	a thermal parameter = $(3\lambda + 2\mu)\alpha_t$
κ	thermal conductivity of the medium
ρ	density of the medium
C_e	specific heat per unit mass at constant strain

uncoupled thermo elasticity (UCT). Classical theories of thermo elasticity use a parabolic type of heat equation which predicts an infinite speed of propagation for the thermal wave which, physically, is not admissible. Biot [1] developed the coupled theory of thermo elasticity which couples the mechanical stress and heat strain but the infinite speed of propagation of thermal wave could not be accounted for. Lord and Shulmon [15] modified Fourier heat conduction equation by introducing linear harmonic to the thermal waves and the heat flux rate term to formulate the generalized theory of thermo elasticity, which could explain the finite speed of propagation of heat waves. Introducing the concept of relaxation time of thermo elastic process to the generalized theory of heat conduction equation, Lord and Shulmon [15] gave a hyperbolic type of heat transport equation. Later the hyperbolic heat conduction equation was further modified by Green and Lindsay [16] who introduced the concept of two relaxation times of the thermal process along with the temperature rate, among other constitutive variables. These rigorous theories have been found to be more realistic than the conventional theories and are in good agreement with the physical experiments. Chandrasekharaiah [17] referred to this wavelike thermal disturbance as "second sound". A brief review of different thermo elastic models can be found in the paper of Hetnarski and Ignaczak [18]. Misra et al. [19] have discussed the thermo elastic wave propagation in half space using non classical thermo elastic model. Sinha and Sinha [20] and Sinha and Elsibai [21] investigated the reflection of thermo elastic waves from the free surface of a solid half-space considering one and two relaxation times, respectively. Abd-alla and Al-Dawy [22] discussed reflection of SV-waves in a generalized thermo elastic medium. Singh [23] considered reflection and transmission of plane waves at a liquid-thermo microstretch interface. Sharma et al. [24] have also discussed the reflection of generalized thermo elastic waves. Singh [25] discussed the problem of reflection of thermo-viscoelastic waves from free surface in the presence of magnetic field. Abd-alla et al. [26] discussed reflection of the generalized magneto-thermo-viscoelastic plane waves. Othman and Song [27] studied the problem of reflection of magneto-thermo elastic waves with two relaxation times and temperature dependent elastic moduli. Sinha and Elsibai [28] considered the problem of reflection and refraction of thermo elastic waves at an interface of two semi-infinite media with two relaxation times. Kumar and Sarathi [29] studied Reflection and refraction of thermo elastic plane waves at an interface between two thermo elastic media without energy dissipation. But they did not consider the presence of initial stress in the medium although, besides the initial stress, temperature of the medium also plays an important factor in the reflection and refraction of elastic waves. The present investigation is concerned with the reflection and refraction of thermo elastic plane waves in a thermo elastic medium without energy dissipation, in presence of initial stress.

2. Definition of the problem

We consider a plane interface separating two solid half-space both of which are homogeneous, isotropic elastic and at uniform absolute temperature T_0 in the undisturbed state. Both the media are in a state of initial stress and the initial stress is different in the two media. A plane SV-wave is incident in medium M at the plane interface and is partially reflected as one SV-wave (rotational wave), one P-wave (dilatational wave) and one thermal wave (dilatational wave). Rest of the wave continues to travel in the other medium M′ after refraction, as one SV-wave, one P-wave and one thermal wave (Fig. 1). We would like to calculate the amplitude ratio of the three reflected components and three transmitted components.

3. Formulation of the problem

We consider a fixed rectangular Cartesian coordinate system $oxyz$ with origin 'o' on the plane $y = 0$. Since we consider a two-dimensional problem, we restrict our analysis to plane strain parallel to oxy-plane. Hence all the field variables depend only on space coordinates x, y and time t and are independent of coordinate z.

For easy reference we follow a convention: All quantities in medium M are represented unprimed whereas corresponding quantities in medium M' are represented as primed.

The initial stress components in medium M and M' are shown in Fig. 2(a) and (b).