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Improved results on robust stability of neutral systems with mixed time-varying delays and nonlinear perturbations ☆

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ABSTRACT

This paper considers the problem of robust stability of neutral systems with mixed time-varying delays and nonlinear perturbations. Two type uncertainties such as nonlinear time-varying parameter perturbations and norm-bounded uncertainties have been discussed. Based on the new Lyapunov–Krasovskii functional with triple integral terms, some integral inequalities and convex combination technique, a new delay-dependent stability criterion for the system is established in terms of linear matrix inequalities (LMIs). Finally, four numerical examples are given to illustrate the effectiveness and an improvement over some existing results in the literature with the proposed results.

1. Introduction

During the last two decades, time-delays have been greatly considered for dynamical systems. Time-delay often appears in many physical systems such as aircraft, chemical, hydraulic, metallurgical processing systems. Unlike ordinary differential equations, time-delayed systems are infinite dimensional in nature. Existence of time-delay, in many cases lead a source of instability and poor performance often appear in many dynamic systems, see for example biological systems, chemical systems and electrical networks. Recently fruitful results on stability analysis for delayed dynamic systems have been investigated by several authors, see the related literature [1–8].

A neutral time delay system contains delays both in its state, and in its derivatives of state. Such system can be found in many places including population [9], distributed networks containing lossless transformation lines [10], heat exchangers, robots in contact with rigid environments [11], etc. Recently, results on stability analysis of neutral system have been investigated Refs. [12–19]. In practice, the systems almost contain some uncertainties because it is very difficult to obtain an exact mathematical model due to environment noise, uncertain or slowly varying parameters, etc. Therefore, robust stability analysis for neutral systems with nonlinear perturbations have been widely investigated by many researchers [20–34]. Additionally, the parametric uncertainties are assumed to be norm bounded and the delay is assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available, see for example [35] and reference therein. In general, the delay-dependent stability criterion is less conservative than delay independent one when the size of time delay is small. In [4], by using convex combination technique, H_∞ filtering for linear neutral systems with mixed time-varying delays and nonlinear perturbations is investigated. An improved delay-dependent criterion

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for asymptotic stability of uncertain dynamic systems with time-varying delays is discussed in [5]. Recently, the triple integral forms of Lyapunov–Krasovskii functional [36,37] for stability of time-varying delays were proposed and showed its improvement of maximum delay bounds. Inspired by the works of [36,37], in this paper, new Lyapunov–Krasovskii functional with triple integral terms involving lower and upper bounds of interval time-varying delays such as $2h_2^2\int_{-h_2}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)S_1\dot{x}(s)dsd\lambda d\theta$ and $2\left[h_2^2-h_1^2\right]\int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)S_2\dot{x}(s)dsd\lambda d\theta$, have been introduced which play an important role in further reduction of conservativeness. This idea motivates this study.

In this paper, we contribute to the improvement of the robust stability analysis of neutral systems with mixed time-varying delays and nonlinear perturbations. By constructing a new Lyapunov–Krasovskii functional with triple integral terms for interval time-varying delays, introducing free-weighting matrices and using convex combination technique, sufficient conditions are derived for the considered systems in terms of LMIs, which can be easily calculated by using Matlab LMI control toolbox. Numerical examples are given to illustrate the effectiveness and less conservatism of the proposed method.

2. Problem description and preliminaries

Consider the following neutral system with mixed time-varying delays and nonlinear perturbations

$$\dot{x}(t)=Ax(t)+A_dx(t-\tau(t))+C\dot{x}(t-h(t))+f_1(x(t),t)+f_2(x(t-\tau(t)),t)+f_3(\dot{x}(t-h(t)),t), \tag{1}$$

$$x(\theta)=\phi(\theta),\quad \dot{x}(\theta)=\varphi(\theta),\quad \forall\theta\in[-\max\{\bar{h},h_2\},0],$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $A,A_d,C\in\mathbb{R}^{n\times n}$ are constant matrices. $h(t)$, $\tau(t)$ are neutral delay and time-varying discrete delay respectively, and they are assumed to satisfy

$$0\leqslant h(t)\leqslant\bar{h},\quad \dot{h}(t)\leqslant\eta<1, \tag{2}$$

$$h_1\leqslant\tau(t)\leqslant h_2,\quad \dot{\tau}(t)\leqslant\mu, \tag{3}$$

where $\bar{h},h_1,h_2,\eta$ and μ are constants. $\phi(\cdot)$, $\varphi(\cdot)$ are the initial functions that are continuously differentiable on $[-\max\{\bar{h},h_2\},0]$.$f_1(x(t),t)$,$f_2(x(t-\tau(t)),t)$,$f_3(\dot{x}(t-h(t)),t)$ are unknown nonlinear perturbations satisfying $f_1(0,t)=0$, $f_2(0,t)=0$, $f_3(0,t)=0$ and

$$\begin{cases} f_1^T(x(t),t)f_1(x(t),t)\leqslant\alpha^2x^T(t)x(t),\\ f_2^T(x(t-\tau(t)),t)f_2(x(t-\tau(t)),t)\leqslant\beta^2x^T(t-\tau(t))x(t-\tau(t)),\\ f_3^T(\dot{x}(t-h(t)),t)f_3(\dot{x}(t-h(t)),t)\leqslant\gamma^2\dot{x}^T(t-h(t))\dot{x}(t-h(t)), \end{cases} \tag{4}$$

where $\alpha\geqslant0$, $\beta\geqslant0$ and $\gamma\geqslant0$ are given constants, for simplicity, we denote $f_1:=f_1(x(t),t)$,$f_2:=f_2(x(t-\tau(t)),t)$,$f_3:=f_3(\dot{x}(t-h(t)),t)$.

In this paper, it is assumed that the uncertainties are of the form

$$[\Delta A(t)\ \Delta A_d(t)\ \Delta C(t)]=LF(t)[E_1\ E_2\ E_3], \tag{5}$$

where L, E_1, E_2 and E_3 are given matrices with appropriate dimensions and $F(t)$ is unknown matrix function satisfying

$$F^T(t)F(t)\leqslant I. \tag{6}$$

Lemma 2.1 (Schur Complement). *Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T=\Omega_1$ and $\Omega_2^T=\Omega_2>0$, then*

$$\Omega_1+\Omega_3^T\Omega_2^{-1}\Omega_3<0,$$

if and only if

$$\begin{bmatrix}\Omega_1 & \Omega_3^T\\ * & -\Omega_2\end{bmatrix}<0\quad\text{or}\quad\begin{bmatrix}-\Omega_2 & \Omega_3\\ * & \Omega_1\end{bmatrix}<0.$$

Lemma 2.2 [5]. *For any scalar $\tau(t)\geqslant0$ and any constant matrix $Q\in\mathbb{R}^{n\times n}$, $Q=Q^T>0$, the following inequality holds:*

$$-\int_{t-\tau(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)ds\leqslant\tau(t)\xi^T(t)VQ^{-1}V^T\xi(t)+2\xi^T(t)V[x(t)-x(t-\tau(t))],$$

where

$$\xi^T(t)=\left[x^T(t)\ x^T(t-\tau(t))\ x^T(t-h_1)\ x^T(t-h_2)\ \dot{x}^T(t)\ \dot{x}^T(t-h(t))\ \left(\int_{t-h_2}^{t}x(s)ds\right)^T\left(\int_{t-h_2}^{t-h_1}x(s)ds\right)^T\left(\int_{t-h(t)}^{t}\dot{x}(s)ds\right)^T f_1^T\ f_2^T\ f_3^T\right]$$

and V is free-weighting matrix with appropriate dimensions.