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# The effect of gravity on flow past a semi-circular cylinder with a constant pressure wake

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**Abstract**

The flow past a semi-cylinder with a trailing wake region is considered. In the absence of gravity the only known high Reynolds number solutions have tangential separation from the body and a cusped shape at the back of the wake. This flow can be a simple model for several situations, including the classical approximation of a constant pressure wake and the flow past an object with a region of trapped fluid of different density (or an air cavity) attached on the downstream side. Here we relax the assumption of high flow speeds to examine the effects of gravity. It is shown that there are situations in which a stagnation point can form either on the body or at the tail of the wake and that there is a minimum velocity beneath which a cavity will not form. Non-uniqueness in the parameter space is found in certain cases.


*Keywords:* Cavity flow; Jets; Free-surface flow

## 1. Introduction

The flow of water past a solid object generates a turbulent wake, the form of which depends on the shape of the object and the speed and characteristics of the liquid. These flows are very difficult to model accurately due to their complicated nature, and it is only recently that digital computers have been able to satisfactorily resolve such flows using numerical methods to obtain solutions to the full Navier–Stokes equations.

Major simplifications in fluid dynamics problems are available if the fluid can be assumed to be inviscid and incompressible and the flow to be irrotational, i.e. high Reynolds number flow. Therefore, in order to make some progress on this problem there have been two simplified model types introduced to consider such flows and their properties. The more complicated of these are known as Prandtl–Bachelor flows, in which a dividing streamline separates a trapped wake region of uniform vorticity from an irrotational outer flow. Some examples of this work can be found in [1–8] and references therein.

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The simpler model assumes the fluid in the wake region to be at constant pressure and to be separated from the outer flow by a dividing streamline. This leads to the same equations as if there was an air cavity (or a region of stagnant fluid with different density) attached to the back of the object, and it is this case that we consider in this paper.

The defining parameter in this problem is the cavitation number

$$K = \frac{p_\infty - p_A}{\frac{1}{2}\rho U^2}, \tag{1}$$

where $\rho$ is the density of the fluid, $U$ is the stream velocity, and $p_\infty$ and $p_A$ are the pressure in the far field and within the cavity, respectively. The name is derived from the possibility of cavitation, but this can only happen if $K > 0$, and no such solutions have been found for this problem (see [9,10]).

Another non-dimensional parameter of significance is the Froude number

$$F = \frac{U}{\sqrt{gR}}, \tag{2}$$

where $R$ is the radius of the cylinder, which represents the effects of the flow speed relative to gravity. Early work on this topic was performed in the infinite Froude number limit and hence $F$ does not appear in these solutions. This assumption implies that since the flow must be fast for a cavity to form the effects of gravity are minimal.

Using this limit and making some geometric assumptions such as thin bodies, solutions have been obtained via the application of free streamline theory and asymptotic techniques, see e.g. [11,10,12–14]. In three dimensions, Cumberbatch and Wu [15] and Haese [16] have calculated such wakes making slender body assumptions. Vanden Broeck [17–19] has recently written a series of papers using numerical methods to investigate the effects of surface tension on the free streamline at the edge of such an air cavity.

In this paper, we extend this work by investigating the influence of the assumption of high flow speed by computing solutions to the flow past a circular cylinder as the speed drops, i.e. as the effect of gravity becomes significant. In the infinite Froude number limit the dividing streamline between the cavity (or wake) and the outer flow must detach from the body and close off tangentially. As the flow slows this condition no longer applies and there is the possibility of stagnation points on the free surface either at separation or reattachment. Furthermore, the orientation of the flow becomes important as gravity will act in different directions. While vertical flows (aligned with the direction of gravity **g**) remain symmetric about the centre line of the cylinder, horizontal flows no longer have this property and so we restrict our attention to flows past half-cylinders where gravity is acting perpendicular to the flow direction. This leads to the four cases shown in Fig. 1.

Vertical flows (Cases I and II) are like a cylindrical pipe situated in a vertical upwash or downflow or plunging downward or upward. If the flow is horizontal then we might consider it to be the flow past a half buried

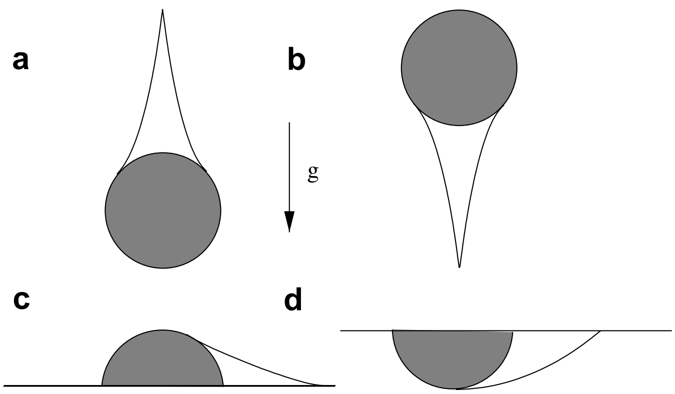


Fig. 1. The four cases to be considered. In each picture gravity is acting downward and the direction of flow is from the opposite direction to the indicated wake region. (a) Case I – vertical flow of a plunging cylinder, (b) Case II – vertically rising cylinder, (c) Case III – flow over a submerged semi-cylinder, (d) flow beneath a semi-cylindrical boom at the surface.