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Numerical modelling of unsteady convective–diffusive heat transfer with a control volume hybrid method

N.J. Rodríguez [a,*], K. Davey [b], J.A. Vázquez Feijoo [c], A. Juárez-Hernandez [d]

[a] *Instituto Tecnológico de Tuxtla Gutiérrez, Department of Mechanical Engineering, 29050, Chiapas, Mexico*
[b] *The University of Manchester, P.O. Box 88, Sackville Street, Manchester M60 1QD, UK*
[c] *Centro Interdisciplinario de Investigación y Desarrollo Integral Regional, IPN, Oaxaca, Mexico*
[d] *Facultad de Ingeniería Mecánica y Eléctrica, Universidad Autónoma de Nuevo León, NL, Mexico*

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Abstract

Presented in this paper is a numerical methodology for the solution of the parabolic governing partial differential equation that describes unsteady advection–diffusion heat transfer. The formulation presented here is shown to be free from the numerical oscillation commonly associated with advection–diffusion heat transfer regardless of the value of the Peclet number. The formulation involves the absorption of the advection term in the unsteady heat equation into the capacitance term. This process is achieved with the use of a control volume methodology applied to each nodal element on a finite-volume mesh. This is shown to ensure that spurious energy losses and gains are avoided and provides for consistency between temperature and energy change. This approach provides unconditional stability and it is shown that good accuracy is achievable with relatively large time-steps.

In order to highlight the features of the approach it is compared against those of benchmark numerical schemes. Detailed analysis is performed for the 1D semi-infinite moving solid problem for which an exact solution is available and for a realistic engineering heat transfer problem. Oscillation free results are achieved at good accuracy for a wide range of Peclet numbers and problems considered.
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1. Introduction

Application of central difference schemes to the advection–diffusion heat transfer equation is known to yield numerical oscillation for values of the Peclet number greater than one [1]. Numerical oscillation appears when one or more algebraic coefficients of the governing finite difference equation changes sign. In order to

* Corresponding author. Address: CIATEQ S.A., Av. Manantiales 23-A, Parque Industrial Bernardo Quintana, El Marqués, 76246 Querétaro, Mexico. Tel.: +52 (442) 1961500; fax: +52 (442) 2215243.
E-mail address: juarez1@hotmail.com (N.J. Rodríguez).

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Nomenclature

c^*	material heat capacitance (J/kg °C)
k^*	material heat conductance (W/m °C)
q^*	modified heat source
t	time (s)
θ	parameter employed in time steeping
ρ	density (kg/m^3)
k	material conductivity or true diffusion (W/m °C)
v	velocity (m/s)
Δx	mesh density
Δt	time steeping
F	strength of advection (kg/(m^2 s))
D	diffusion conductance (W/(m^2 °C))
Pe	Peclet number (the ratio between F and D)
a_{E}^{n+1}	the value of the east coefficient at time $n+1$
a_{W}^{n+1}	the value of the west coefficient at time $n+1$
a_{E}^{n}	the value of the east coefficient at time n
a_{W}^{n}	the value of the west coefficient at time n
a_{P}^{n}	coefficient with boundary condition and transient term at time n
T_{b}	temperature at the boundary
T_{∞}	temperature at ∞
T_0	initial temperature

Subscripts

f	face of the control volume
e	east face of one-dimensional control volume
w	west face of one-dimensional control volume
E	east grid point
W	west grid point
P	at the unknown point

prevent numerical instabilities a number of alternative difference equations have been proposed. One of the first attempts to solve this problem involved adding "artificial diffusion terms" to correct the under-diffusive character of the Galerkin finite element formulation and the analogous central finite difference schemes [2]. The relationship of this approach with the upwind finite difference method leads to a variety of Petrov–Galerkin methods [2]. Petrov–Galerkin methods can be interpreted as extensions of the standard Galerkin variational form of the finite element method (FEM) by adding residual-based integral terms computed over each element domain. This is the case of the Upwind FEM [3,4], the Streamline Upwind Petrov–Galerkin (SUPG) method and the pressure stabilizing/Petrov–Galerkin (PSPG) formulation for incompressible flows which are some of the most prevalent stabilized methods [5–9].

All the methods above make use of a single stabilization parameter which suffices to stabilize the numerical solution along the velocity (streamline) direction. The computation of this stabilization parameter typically involves the measure of a local length scale and other quantities such as element Reynolds and Courant numbers. Stabilization parameters are typically denoted as τ in the literature [10,11]. The SUPG method, for example, which has successfully been applied to many different situations, involves the addition of a consistent term that provides "additional diffusion" in the streamline direction [4,6]. The amount of such "additional diffusion" is tuned by the parameter τ and accurate results are particularly sensitive to this. Several recipes have been proposed for the choice of τ [12]. Although the approach has been proved to have a solid mathematical basis in several cases of practical interest [13], a suitable convincing argument to guide the choice of τ is still

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