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Stokes' first problem for a micropolar fluid through state-space approach

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Abstract

The flow of an incompressible micropolar fluid over a suddenly moved plate is considered under isothermal conditions. State-space technique is used to find the solution of the problem. Inversion of Laplace transform is carried out using a numerical approach. The variation of velocity and microrotation fields is studied with respect to various flow parameters and the results are presented through graphs.

Keywords: Micropolar fluid; State-space technique; Stokes' first problem; Laplace transform; Numerical inversion

1. Introduction

Four decades have passed ever since the theory of micropolar fluid was initiated by Eringen [1] in 1966. It is well known that in many of the real fluids the shear behavior cannot be characterized by Newtonian relationships and hence researchers have proposed diverse non-Newtonian fluid theories to explain the deviation in the behavior of real fluids with that of Newtonian fluids. One such theory is that of micropolar fluids. This theory accounts for the internal characteristics of the substructure particles with the assumption that they are allowed to undergo rotation independent of their linear velocity. Micropolar fluids represent fluids consisting of rigid randomly oriented particles suspended in a viscous medium when the deformation of the particles is ignored. This constitutes a substantial generalization of the Navier–Stokes model. Micropolar fluids belong to the class of fluids with non-symmetric stress tensor which are referred to as polar fluids. This is a class which is more general than the one which we face in classical fluid dynamics. The theory of micropolar fluids may form a suitable non-Newtonian fluid model that can be used to analyze the behavior of lubricants, colloidal suspensions, polymeric fluids, liquid crystals and animal blood. The equations of motion characterizing a micropolar fluid flow are non-linear in nature (as in the case of Newtonian viscous fluids) and are constituted by a coupled system of vector differential equations. In any micropolar fluid flow problem, in addition to the

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Nomenclature

(x, y)	space coordinates
u	velocity of the fluid along the x-direction
U	velocity of the plate
c	microrotation component
ρ	density
t	time
p	pressure
j	gyration parameters
$\bar{q}$	velocity vector
$\bar{\upsilon}$	microrotation vector
$\bar{f}$	body forces per unit mass
$\bar{l}$	body couple per unit mass
λ_1, μ, k	viscosity coefficients
α, β, γ	gyro viscosity coefficients
υ_i	components of the microrotation vector
ω_i	components of the vorticity vector
d_{ij}	components of the rate of strain
δ_{ij}	kronecker symbol
t_{ij}	force stress tensor
m_{ij}	couple stress tensor
ε_{ijk}	Levi-Civita symbol or permutation symbol

usual field variables pressure p and velocity vector $\bar{q}$, we come across another field variable called microrotation vector $\bar{\upsilon}$ which is independent of $\bar{q}$. To understand the departure from the viscous fluid flow model, several problems that were studied in viscous fluid flow theory have also been studied in the realm of micropolar fluids. An account of the earlier developments in polar fluid theory can be seen in [2] of V.K. Stokes and the existing state of art can be seen in the excellent treatise of Lukaszewicz [3].

In this paper, we propose to study the Stokes' first problem for a micropolar fluid using state-space approach, which has been used till recently in modern control systems theory. Consider an infinitely long flat plate above which a fluid exists. Initially both the plate and fluid are assumed to be at rest. Let us suddenly impart a constant velocity to the plate in its own plane. Stokes in 1851 and again Raylegh in 1911 have discussed the fluid motion above the plate independently taking the fluid to be Newtonian [4]. In the literature this problem is referred to as Stokes' first problem. Subsequently in 1962 Tanner [5] considered the above problem with Maxwell fluid in place of the Newtonian fluid. Prezoisi and Joseph [6] and Phan-Thien and Chew [7] studied Stokes' first problem for viscoelastic fluids. In recent years many investigators have studied Stokes' first problem for non-Newtonian fluids with different constitutive equations (see Ref. [8–16]). In this paper we study the problem for an incompressible micropolar fluid whose constitutive equations were proposed by Eringen [1]. We solve the problem through the method of state-space formulation which is more general than the classical Laplace transform and Fourier transform techniques. The state-space theory is applicable to all systems that can be analyzed by integral transforms in time and is successfully employed to study, in particular, problems in modern control theory. As Helmy et al. [17] observe, the state-space approach is useful to study linear systems with time varying parameters in essentially the same manner as the time invariant linear systems.

2. Basic equations for incompressible micropolar fluid flow

The field equations of micropolar fluid dynamics are [1],