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An unstructured finite volume time domain method for structural dynamics

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ABSTRACT

An unstructured finite volume time domain method (UFVTDM) is proposed to simulate stress wave propagation, in which the original variables of displacement and stress are solved based on the dynamic equilibrium equations. An Euler explicit and unstructured finite volume method is used for time dependent and spacial terms respectively. The displacements are stored on the cell vertex and a vertex based finite volume method is formed with that integral surface and the stresses are as assumed to be uniform in the cell. The present UFVTDM has several features. (1) The governing equations are discretized with the finite volume method which naturally follows conservation laws. (2) It can handle complex engineering problem. (3) This method is also able to analyze the natural characteristics and the numerical experiment shows that it is very efficient. Several cases are used to show the capability of the algorithm.

1. Introduction

It is well known that the finite element method (FEM) is the most powerful tool for computational structural dynamics (CSD) problems, while the finite volume method (FVM) is the most popular one in computational fluid dynamics (CFD). The FVM may be considered as a particular case of the FEM method with non-Galerkin weighting [1]. However, applications and directions of development have resulted in numerical software tools for CFD and CSD that are different in almost every aspect for their different properties [2]. Therefore it is frustrated in modelling the emerging multi-physics problem of fluid–structure interaction in a consistent manner. This demand for consistency has led to the development of computational techniques that facilitate multi-physics modelling entirely within the context of FEM or FVM [3].

The reason for the unpopularity of the FVM in CSD is that FVM is well known to be less accurate than Galerkin-based finite elements for self-adjoint (elliptic) problems [1]. The essential difference between FEM and FVM in the numerical discretization of the second-order partial differential equations is negligible and in many cases the two methods are almost equivalent [4]. Recently there have been a number of researches to employ FVM to solve various CSD problems. Fryer et al. have shown a control volume procedure for solving the elastic stress–strain equations for two dimensional arbitrarily complex geometries [5]. Demirdžić et al. have employed FVM for prediction of stresses and displacements in thermo-elasto-plastic material [6–8]. Wheel has applied FVM to analyse a benchmark stress concentration problem [9], and even shown FVM achieves better accuracy than FEM for a NAFEMS (National Agency for FE Methods and Standards) steel elliptic membrane benchmark. Wheel has also employed FVM to analyse the bending deformation of thick and thin plates [10]. Bailey et al. [11], Taylor et al. [12] and Hattel et al. [13] have also performed the solution of different CSD problems. Wenke

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adds a rotational variable to improve the accuracy of FVM [14]. Fallah has developed a cell vertex and cell centred forms of FVM for the analysis of transversely loaded plates [15].

Slone and her co-authors have done a great work on numerical method in multi-physics coupling problems and it is very important method to analyze industry engineering problem. They proposed a vertex based finite volume method to analyze the solid dynamics. The formula and solution algorithm of the method are discussed and in which a weighting procedure quite similar as a finite element method is used [3]. The advantage is that it is helpful to uniform the FEM and FVM and make the FVM easy to be accepted by people in computational solid dynamics field, but Slone's method does not have a clearly physical meaning and an explicitly conservative form as general FVM. On the other side, Liu et al. have developed the unstructured grid method to simulate two-dimensional stress wave propagation within elastic structures [16]. This method has mainly applied to seismic waves. We extended this method to 3D case and discussed the boundary conditions. By introducing this unstructured FVM, we can analyze the structural natural characteristics.

CSD problems have been carried out in time domain and frequency domain. The advantages of time domain modelling algorithms are that they are easy to carry out and the requirements on computer memory are not as great as frequency domain modelling algorithms. However, if we want to know the message of frequency domain, the time domain modelling algorithms may consumed more computation time. On the other hand, frequency-domain modelling algorithms require a number of computer core memory because of the storage needed for the complex matrix. Hence, frequency domain modelling algorithms may not be appropriate for large and complicated models, such as 3D models.

In this study, a 3D unstructured finite volume time domain method (UFVTDM) with original variable is presented. The displacement vector is stored on the cell vertex and a vertex based finite volume method is formed where stress tensor is as assumed to be uniform in the cell. The strain tensor can be calculated with the displacement vector on the cell vertex. A time marching algorithm is adopted for displacement vector. This method is implemented in an in-house code GTEA (General Transport Equation Analyzer) which is an unstructured grids finite volume solver.

2. Mathematic model

2.1. Governing equations

We apply Newton's second law on a control volume in computational domain. The following formula holds:

$$\int_V \rho \ddot{\mathbf{u}} dV = \int_V \nabla \cdot \boldsymbol{\sigma} dV \tag{1}$$

where ρ is the material density, $\ddot{\mathbf{u}}$ and $\boldsymbol{\sigma}$ are the acceleration vector and the Cauchy stress tensor respectively, V and dV are the control volume and integral variable.

The constitutive equations for the elastic material are as the following.

$$\sigma_x = \lambda\theta + 2G\varepsilon_x, \quad \sigma_y = \lambda\theta + 2G\varepsilon_y, \quad \sigma_z = \lambda\theta + 2G\varepsilon_z, \quad \tau_{xy} = G\gamma_{xy}, \quad \tau_{zy} = G\gamma_{zy}, \quad \tau_{xz} = G\gamma_{xz}$$
$$\lambda = \frac{\mu E}{(1+\mu)(1-2\mu)}, \quad G = \frac{E}{2(1+\mu)}, \quad \theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

μ is Poisson ratio and E is Young's modulus.

From the geometry condition the following correlations are hold.

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \quad \gamma_{xy} = \gamma_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{zy} = \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

2.2. Numerical methods

The left hand of Eq. (1) can be read in the following form base on a uniform distribution of the acceleration in the control volume.

$$\int_V \rho \ddot{\mathbf{u}} dV = \rho \ddot{\mathbf{u}} V \tag{2}$$

And the Gauss theorem is used on the right term, and we obtain:

$$\int_V \nabla \cdot \boldsymbol{\sigma} dV = \oint_s \boldsymbol{\sigma} \cdot \mathbf{n} ds \tag{3}$$

The stress in the tetrahedron cell 1234 in Fig. 1 is uniform and the cell contribution to control volume around node 1 can be expressed by $\boldsymbol{\sigma}\cdot\mathbf{s}_1$ in which $\mathbf{s}_1$ is the area vector of space polygon $E_{12}O_4E_{13}O_3E_{14}O_2$ which is constructed of three quadrilaterals.

And the following formula is hold.

$$\mathbf{s}_1 = a_1\mathbf{i} + b_1\mathbf{j} + c_1\mathbf{k} \tag{4}$$