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Large amplitude vibration of heated corrugated circular plates with shallow sinusoidal corrugations

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ABSTRACT

This paper presents a nonlinear free vibration analysis of corrugated circular plates with shallow sinusoidal corrugations under uniformly static ambient temperature. Based on the nonlinear bending theory of thin shallow shells, the governing equations for corrugated plates are established from Hamilton's principle. These partial differential equations are reduced to corresponding ordinary ones by elimination of the time variable with Kantorovich method following an assumed harmonic time mode. The resulting equations, which form a nonlinear two-point boundary value problem in spatial variable, are then solved numerically by shooting method, and the temperature-dependent characteristic relations of frequency vs. amplitude for nonlinear vibration of heated corrugated plates are obtained successfully. The comparison with available published results shows that the numerical analysis here is of good reliability. A detailed parametric study is conducted involving the dependency of nonlinear frequency on the depth and density of corrugations along with the temperature change. Effects of these variables on the trend of nonlinearity are plotted and discussed.

1. Introduction

Thin-walled plates and shells are commonly encountered in various branches of modern engineering applications and they are often subjected to severe dynamic loading. This may result in large vibration amplitudes of these structures. Therefore, the geometrically-induced nonlinear vibration problem has to be taken into account for proper design in such situations.

Due to the complex nature of the basic equations governing the vibrations of plates and shells with large amplitude, the closed-form analytical solution is not always available. Hence, each problem has received a special treatment involving some particular approximations. The assumption generally used is the separation of time and space variables [1,2]. After going through the literature available on the topic of nonlinear vibration problems in a plate or shell, it is found that the geometrically nonlinear flexural vibration of plates and shells have been studied by many researchers. Solution methods used for this purpose include the so-called assumed-space-mode method and assumed-time-mode method [3,4]. The former assumes a spatial function which satisfies the related boundary conditions to eliminate the space co-ordinate, thus, the nonlinear partial differential governing equations are reduced to a set of nonlinear ordinary differential equations, with time as an independent variable [5,6]. For example, the finite element method [7,8] and the Galerkin method [9,10] have been to use an assumed space mode. Recent researches carried out on plates vibration using the finite element method can be found in works by Ribeiro and Petyt [11–14], where the vibration of thin plates with geometrical nonlinearity are studied by the

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hierarchical finite element method. In the second approach, the dependence on time is assumed to be harmonic. Then, by utilizing the Kantorovich time-averaging procedure, the nonlinear partial differential governing equations are converted into the corresponding nonlinear ordinary differential equations, which form a nonlinear boundary value problem in spatial variable [15]. The treatment of finding an assumed-time-mode solution then can be numerically studied, for example, by the combined Runge–Kutta method and Newton-Raphson method, which is called the initial value method [15] or shooting method [16–19].

A variety of computational methods have been proposed and adopted by many researchers in the field of large amplitude vibration modeling and analysis for plates and shells. Extensive literature reviews on the nonlinear vibration of plates are given by Chia [20] and Sathyamoorthy [21]; A survey on vibration of shallow shells has been reported by Liew et al. [22]. Among the commonly used approaches, the aforementioned shooting method has been proved to be an efficient method. The present paper extends this method to the nonlinear vibration problem of the heated corrugated circular plate with shallow sinusoidal corrugations.

The corrugated thin plates discussed herein are extensively used in engineering structures. Due to their high strength and large flexibility, the corrugated plates usually experience large amplitude of vibrations. Therefore, it is necessary to understand the effect of geometric nonlinearity on their dynamic behavior.

For the studies on geometrically-induced nonlinearity of corrugated plates, the researchers have given a great attention, but problems involved are quite difficult to solve satisfactorily and adequately because of the complicated geometry of the plates as well as the nonlinear mathematics. Review of the literature indicates that different investigators studied the nonlinear problems in a corrugated circular plate, analytically and numerically, based on two nonlinear bending theories: isotropic shallow shell theory and anisotropic plane plate theory. The consideration in nonlinear bending theory of thin shallow shells was first forwarded by Panov [23] who studied the large deflection problem of shallow corrugated membrane. Axelrad [24] discussed the membrane with deep sinusoidal corrugations using Galerkin's method. Hamada et al. [25] solved the bending problem of a diaphragm with a boundary corrugation using the finite difference method. Bihari and Elbert [26] obtained the deflection and radial displacement of a corrugated plate by directly solving 6 coupled first-order differential equations. Chen [27] solved the large deflection equations of shells for a corrugated circular plate with shallow sinusoidal corrugation using the modified iteration method. Liu and Yuan [28,29] investigated the bending problem of a corrugated plate with a large boundary corrugation under actions of various loads based on the simplified Reissner's equation of axisymmetric shells of revolution by means of the integral equation method. The second theory assumes a corrugated plate as an anisotropic plate and the large deflection theory of thin anisotropic plane plates had been adopted. The pioneer treatment in this theory was suggested by Haringx [30] who developed a new means to transform a corrugated circular plate into equivalent orthotropic circular plate. Employing this theory, Liu and Li [31,32] studied the nonlinear bending and free vibration for corrugated circular plate with or without plane central regions via Galerkin's method and modified iteration method. Wang et al. [33] dealt with the nonlinear free vibration of corrugated circular plates and gained the analytical solutions for the amplitude-frequency relationship through perturbation-variation method, but failed to give numerical results and discussions. Generally, the nonlinear bending theory of plates based approach is applicable to corrugated circular plate with dense corrugations of various types, e.g., toothed corrugations and sinusoidal corrugations, while the shells based way proves to be independent of the density of corrugations [27,34].

Though numerical and analytical treatments of geometric nonlinearity of a corrugated plate have interested many investigators from the view of past literature, most of which have been limit to the problem of large deflection bending, where the plate undergoes various laterally static loadings [35,36]. Some studies were carried out in the large amplitude vibration of these plates, however, due to the more complex nature of resulting governing equations when time variable is involved, only limited attention has been given. What is more, very few attempts have been made to predict the large amplitude vibration behavior of corrugated circular plates in an environment of changing temperatures to the best of authors' knowledge [37].

The current work focuses on the axisymmetrical nonlinear vibration of a uniformly heated corrugated circular plate with shallow sinusoidal corrugations. Here, the temperature is time-independent. The corrugated plate is assumed to be a thin plate with small axisymmetric initial deflection, and the partial differential equations governing the nonlinear free vibration of heated circular corrugated plates are formulated from Hamilton's principle in terms of Von Kármán's theory. Assuming the existence of harmonic vibration, the time variable is eliminated by means of the Kantorovich time-averaging method [15–19]. The governing equations thus reduce to a pair of nonlinear ordinary differential equations with two-point boundary conditions. A numerical study is then accomplished by shooting method. Several numerical results for heated corrugated circular plates are presented in both tabular and graphical forms, which demonstrate the accuracy of present method and illustrate the characteristic relations between the frequency and amplitude under various parameters of ambient temperature and plate geometry.

2. Dynamic governing equations

Consider a concentric circular corrugated plate with full shallow sinusoidal corrugations having an outer radius a, constant thickness h, corrugation semi-wave length l and wave height f. Let (r,θ,z) denote a set of cylindrical co-ordinates, as shown in Fig. 1, then the wave of plate can be expressed as