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Implicit–explicit predictor–corrector schemes for nonlinear parabolic differential equations ☆

Dongfang Li [a], Chengjian Zhang [a,*], Wansheng Wang [b], Yangjing Zhang [c]

[a] School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, China
[b] School of Mathematics and Computational Science, Changsha University of Science and Technology, Yuntang Campus, Changsha 410114, China
[c] Foreign Language School, Huazhong Agriculture University, Wuhan 430070, China

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ABSTRACT

In the present paper, a family of predictor–corrector (PC) schemes are developed for the numerical solution of nonlinear parabolic differential equations. Iterative processes are avoided by use of the implicit–explicit (IMEX) methods. Moreover, compared to the predictor schemes, the proposed methods usually have superior accuracy and stability properties. Some confirmation of these are illustrated by using the schemes on the well-known Fisher's equation.

1. Introduction

Nonlinear parabolic problems (e.g., Fisher's equation, Huxley equation) frequently arise in many scientific areas and intrigue lots of researchers in computational implementation and numerical analysis (e.g., [1–12]). For the numerical solution of the parabolic differential equation:

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(t,x,u), \quad (x,t) \in [0,X] \times [0,\infty), \\ &u(x,0) = u_0(x), \quad u(0,t) = \alpha(t), \quad u(X,t) = \beta(t), \end{aligned} \tag{1.1}$$

where t and x denote the time and spatial coordinate, respectively, $u_0(x)$, $\alpha(t)$ and $\beta(t)$ are given specified initial and boundary conditions, $f(t,x,u)$ is a nonlinear Lipschitz continuous with the conditions to ensure that the nonlinear parabolic equation (1.1) owns a unique solution, we discretize in space with stepsize $h = X/N$ and approximate the second-order spatial derivative by the central difference operator:

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2},$$

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* Corresponding author.
E-mail addresses: lidongfang1983@gmail.com (D. Li), cjzhang@mail.hust.edu.cn (C. Zhang).

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where $u_{i,j}$ is a numerical approximation to $u(x_i, t_j)$. Then, Eq. (1.1) has been converted to an ordinary differential equation (ODE) in the form of:

$$\frac{du}{dt} = Au + g(t,u), \quad u(0) = u_0(t). \tag{1.2}$$

Here

$$A = \frac{1}{h^2}\begin{bmatrix} -2 & 1 & \cdots & 0 & 0 \\ 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & \cdots & 1 & -2 \end{bmatrix}_{(N-1)\times(N-1)}$$

with its eigenvalues $\lambda_j^A = \frac{1}{h^2}[-2 + 2\cos(j\pi/N)], j = 1, 2, \ldots, N-1$.

Obviously, a chosen small space-size h gives rise to a matrix with a large condition number. This yields an inherently stiff ODE. In terms of the ODE, the implicit linear multistep and Runge–Kutta methods are favorable schemes for previous investigators to avoid a small time step-size (see e.g., [12–16]), whereas, these two schemes generate the nonlinear algebraic equations, a problem needed to be solved by Newton iterative processes. Considering that the update of the Jacobian matrix at every time step within the Newton iterative process is costly, the two schemes are undesirable to some extent.

In addition, PC schemes are also a family of effective methods, which are successfully applied in the solution for time-dependent partial differential equations. In [17], Jacques constructed PC^m methods corresponding to $P(EC)^m$ methods for parabolic problems. Convergence was achieved with a prescribed number of iterations. Based on split Adams–Moulton formulas, Voss and Casper [18] revealed to us a novel family of PC schemes for stiff ODEs. These schemes owned higher order and smaller error constant than corresponding split backward differentiation formulas. And a modified Newton iteration scheme was used at every time step. Later, Voss and Khaliq [19] considered the θ-methods in a linearly implicit form as the predictor. They derived an implicit second-order PC scheme for (1.1). Recently, split-predictor and split-corrector iterations were introduced by Celnik et al. [20] for stiff problems. Successive over-relaxation was used to prevent divergence in nonlinear systems for some large step sizes. However, few researchers attempt to find some more accurate PC methods excluded from the iterative methods in the numerical solution to the stiff ODE (1.2).

Since the ODE (1.2) contains additive terms with different stiffness properties, we can make full use of these features through the application of special numerical schemes. In the present paper, the adoption of IMEX methods as the predictor and implicit schemes as the corrector generates our PC algorithm. Meantime, iterative methods become dispensable when a transformation of the discrete approximation is given. Moreover, our PC method possesses extraordinary accuracy and wider stability regions than the previously mentioned IMEX algorithm. And all the obtained theoretical results are confirmed by numerical tests on a fisher's equation.

The rest of paper is structured as follows. In Section 2, we derive classes of IMEX PC schemes for Eq. (1.2). Section 3 is devoted to discuss the stability properties of the methods. In Section 4, with some numerical tests, we confirm the methods' effectiveness and the obtained theoretical results. Finally, conclusions and discussions for this paper are summarized in Section 5.

2. Predictor–corrector schemes

The strategy we adopt for the predictor is IMEX schemes, which were first introduced for time dependent partial differential equations by Ascher et al. in [21]. Let k be the discretized time stepsize and u_n denote the approximation solution at $t_n = kn$. Applying s-step IMEX scheme to (1.2) gives:

$$\sum_{j=0}^{s} a_j u_{n+j} = k\sum_{j=0}^{s} b_j A u_{n+j} + k\sum_{j=0}^{s-1} c_j g(t_{n+j}, u_{n+j}), \tag{2.1}$$

where $b_s \neq 0$.

We take a transformation for (2.1) in our actual computation. This yields our following PC methods:

the predictor:

$$(a_s I - b_s kA)\tilde{u}_{n+s} = \sum_{j=0}^{s-1}\left(-a_j u_{n+j} + kb_j A u_{n+j} + kc_j g(t_{n+j}, u_{n+j})\right), \tag{2.2}$$

the corrector:

$$(a_s I - b_s kA)u_{n+s} = \sum_{j=0}^{s-1}\left(-a_j u_{n+j} + kb_j A u_{n+j} + kb_j g(t_{n+j}, u_{n+j})\right) + kb_s g(t_{n+s}, \tilde{u}_{n+s}). \tag{2.3}$$