



Numerical simulation of a two-phase melt spinning model

Renu Dhadwal^{*}

Center for Mathematical Modeling, FLAME, Pune, India

ARTICLE INFO

Article history:

Received 16 September 2009

Received in revised form 5 December 2010

Accepted 9 December 2010

Available online 17 December 2010

Keywords:

Melt spinning

Crystallization

Necking

Giesekus model

Viscoelasticity

Two-phase model

ABSTRACT

We consider a two-phase model of melt spinning including flow induced crystallization. Introducing slight modifications in the model we perform numerical simulations on it. We present comparison of our velocity profiles with the experimental profiles provided by the company Freudenberg & Co.

© 2010 Elsevier Inc. All rights reserved.

1. Introduction

The main objective of several textile industries has been the production of long, thin polymer fibres. Melt spinning is an important industrial process used in the manufacture of synthetic fibres such as Nylon. The problem of melt spinning has been an area of interest for engineers, physicists, chemists and mathematicians alike since the past 50 years. Search for faster and more reliable manufacturing techniques, the demand for better quality and low cost-products and enormous progress in the field of rheology of polymeric fluids has initiated interest in the theoretical study of various manufacturing processes like the melt spinning process. Melt spinning is a process in which molten polymer is extruded from a pressurised reservoir through a small circular orifice called the spinneret. The liquid jet undergoes stretching, cooling and solidification. The solidified filament is wound up via a take-up device at a higher speed than the extrusion velocity to ensure that the fibre is stretched. The fibre is then subjected to other processing steps.

High speed, non-isothermal spinning is associated with a concentrated neck like deformation process (where the diameter of the fibre shows a sudden, sharp decrease forming a neck-like region) and the development of high tensile stresses which result in the so called flow-induced crystallization. As a result, the spun fibres can be vastly superior in quality, possessing better mechanical and transport properties.

To take into account effects of crystallization and to understand the necking phenomena better, it has been inevitable for engineers to come up with models where there is a coupling between the continuum equations and the microstructure. One work was by Doufas, Mchugh and Miller [1,2] wherein the Giesekus model has been used to model nonlinear viscoelastic effects. The model consists of two ordinary differential equation (ODE) systems which are coupled by boundary conditions at the interface. This gives rise to a free boundary value problem where the point of onset of crystallization (free boundary) is unknown and a part of the solution. One ODE system describes the melt spinning equations before the onset of

^{*} Tel.: +91 9318759713.

E-mail addresses: renu.dhadwal@flame.edu.in, renu.dhadwal29@gmail.com

crystallization and we abbreviate this system as BOC. The other system describing the phase after the onset of crystallization will be called AOC.

The purpose of this paper is to perform simulations on this highly complex model and to apply the model to a specific industrial application. The paper is organised as follows. The first and second sections describe the model briefly and much of the material contained therein has been taken from [1] unless otherwise specified. In Section 3, we discuss the numerical scheme used to do our simulations. Section 4 discusses the results of our simulations and in particular the comparison of our velocity profiles with those provided by the company *Freudenberg & Co.* We conclude the paper with brief remarks in Section 5.

2. Governing equations

The governing equations of this model consist of transport equations (macroscopic equations) and the viscoelastic constitutive model in the microscopic form (in terms of conformation tensors as opposed to stress tensors). These conformation tensors describe the configuration of the polymer molecules in the amorphous phase. Once the onset of crystallization takes place the polymer molecules are modeled as rigid rods and their configuration is given by an orientation tensor $\mathbf{S}$. The assumptions in the model are that the quantities of interest are cross-sectionally averaged (1-d equations) and cylindrical coordinate system is employed to describe the evolution of equations. We consider only the steady state equations. A detailed derivation of the transport equations can be found in [3]. Here we merely give the final equations got after integrating the equations of conservation of mass, momentum and energy over the cross section of the fibre.

$$W = \frac{\pi \rho v_z D^2}{4}, \quad (1)$$

$$W \frac{dv_z}{dz} = -\frac{C_d}{2} \rho_a D (v_z - v_d)^2 + \frac{\pi s}{2} \frac{dD}{dz} + \frac{d}{dz} A (\tau_{zz} - \tau_{rr}) + A \rho g, \quad (2)$$

$$\rho C_p v_z \frac{dT}{dz} = \frac{-4}{D} k (T - T_a) + (\tau_{zz} - \tau_{rr}) \frac{dv_z}{dz} + \rho \Delta H v_z \frac{d\phi}{dz}. \quad (3)$$

Here W denotes the mass throughput per unit time, v_z is the axial velocity of the fibre, D the diameter of the fibre and ρ density. Therefore the first equation simply states that the mass throughput is constant along the spinline. In the second equation, the force due to air drag has been taken into account which represents the first term on the right hand side with C_d representing the air drag coefficient, v_d the downward component of the air velocity, and ρ_a the density of air. In the second term, s denotes the surface tension, τ_{zz} and τ_{rr} in the third term represent the two diagonal stress components. The fourth term describes the force due to gravity in the momentum equation. *Here we already introduce our first modification in the momentum equation. In the original model a linear term has been used to model the force due to air drag which employs a Bingham number. For our purposes Bingham number is not suitable. Bingham plastics require yield stress to deform. The fibres under consideration are viscoelastic that do not require yield stress to deform. Therefore we have employed an alternative air drag term with a quadratic velocity term.*

Before the onset of crystallization the polymer is amorphous. The polymer melt in its molten state can be thought of as consisting of n nonlinear elastic dumbell molecules. Each of the molecule chains is assumed to consist of N_0 structural units or statistical links of length l . Let $\mathbf{R}$ denote the end-to-end distance vector between the two end points of a chain. The tensor $\mathbf{c} = \langle \mathbf{R} \otimes \mathbf{R} \rangle$ represents the configuration of the melt, [4]. The kinematics of polymer molecules and their rheology is described in details in [4,5]. The rheological behaviour of the melt is modeled by a modification of the single-mode Giesekus model, [6]:

$$\dot{\mathbf{c}} = -\frac{4k_b T}{\zeta} \left((1 - \alpha) \mathbf{I} + \alpha \frac{K}{k_b T} \mathbf{c} \right) \cdot \left(\frac{K}{k_b T} \mathbf{c} - \mathbf{I} \right).$$

Here, ζ denotes the hydrodynamic drag between the beads of the molecules. To take into account the finite extensibility of chains, a modified spring law has been used, [7]. By defining $\lambda_a = \zeta/4K$ the final evolution equation for the conformation tensor is obtained

$$\dot{\mathbf{c}} = -\frac{1}{\lambda_a(T)} \frac{k_b T}{K} \left((1 - \alpha) \mathbf{I} + \alpha \frac{K}{k_b T} E \mathbf{c} \right) \cdot \left(\frac{K_0}{k_b T} E \mathbf{c} - \mathbf{I} \right), \quad (4)$$

where k_b is the Boltzmann constant, K is the Hookean spring constant, λ_a is the characteristic relaxation time of the melt and E is the non-linear spring force factor given by $E = L^{-1}(e)/3e$ where L^{-1} is the inverse Langevin function and $e = (\text{tr} \mathbf{c})^{1/2}/N_0 l$. Expanding the convected derivative term and considering only the steady state so that partial derivatives with respect to time vanish, the following evolution equations for the components of the conformation tensor c_{zz} and c_{rr} are obtained:

$$v_z \frac{dc_{zz}}{dz} = 2c_{zz} \frac{dv_z}{dz} - \frac{1}{\lambda_a(T)} \frac{k_b T}{K} \left(1 - \alpha + \alpha \frac{K}{k_b T} E c_{zz} \right) \left(\frac{K}{k_b T} E c_{zz} - 1 \right), \quad (5)$$

$$v_z \frac{dc_{rr}}{dz} = -c_{rr} \frac{dv_z}{dz} - \frac{1}{\lambda_a(T)} \frac{k_b T}{K} \left(1 - \alpha + \alpha \frac{K}{k_b T} E c_{rr} \right) \left(\frac{K}{k_b T} E c_{rr} - 1 \right). \quad (6)$$

Download English Version:

<https://daneshyari.com/en/article/1706117>

Download Persian Version:

<https://daneshyari.com/article/1706117>

[Daneshyari.com](https://daneshyari.com)