Available online at www.sciencedirect.com

ScienceDirect

Applied Mathematical Modelling 32 (2008) 2775–2791

APPLIED MATHEMATICAL MODELLING

www.elsevier.com/locate/apm

Self-adaptive FEM numerical modeling of the mild-slope equation

Shu-Xue Liu *, Bing Sun, Zhong-Bin Sun, Jin-Xuan Li

State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China

Received 1 March 2006; received in revised form 1 July 2007; accepted 20 September 2007
Available online 12 October 2007

Abstract

Based on the linear wave theory, the mild-slope equation (MSE) is a preferred mathematical model to simulate near-shore wave propagation. A numerical model to solve the MSE is developed here on the basis of a self-adaptive finite element model (FEM) combined with an iterative method to determine the wave direction angle to the boundary and thus to improve the treatment of the boundary conditions. The numerical resolution of the waves into ideal domains and multi-directional waves through a breakwater gap shows that the numerical model developed here is effective in representing wave absorption at the absorbing boundaries and can be used to simulate multidirectional wave propagation. Finally, the simulated wave distribution in a real harbor shows that the numerical model can be used for engineering practice.

Keywords: Wave direction angle; Self-adaptive method; Numerical model; FEM method; Mild-slope equation

1. Introduction

Nearshore wave propagation is a complex process combining wave refraction, diffraction, and reflection. Accurately simulating wave propagation is very important for determining design waves for coastal engineering problems. Among several linear theories, the mild-slope equation (MSE) derived by Berkhoff [1] provides a steady-state solution for linear regular waves over a wide range of water depths and is still one of the most effective mathematical models available to describe combined wave effects. Some efforts have been made to extend the application range of the equations by the inclusion of a dissipation factor that takes bottom friction and wave breaking into consideration. Typical references include Chen [2], Kostense et al. [3], Battjes and Janssen [4], and Dally et al. [5]. A detailed review of work on the MSE can be found in Dingemans [6]. In addition, Rojanakamthorn et al. [7] and Losada et al. [8,9] extended the basic mild-slope equation solutions to include analytical derivations for breaking and nonbreaking waves traveling over an arbitrary finite porous bottom. Silva et al. [10] presented a derivation of the mild-slope equation which retains second-order terms

* Corresponding author. Fax: +86 411 84708526.
E-mail address: liusx@dlut.edu.cn (S.-X. Liu).

0307-904X/$ - see front matter
doi:10.1016/j.apm.2007.09.033

and extended this solution to consider the effects of evanescent modes. Silva et al. [11] showed that these solutions are valid for both exposed and submerged porous structures.

Because numerical models are powerful tools to assist in estimating wave properties in harbors or wave shoaling, many numerical models based on the MSE have been developed, for example by Bettess and Zienkiewicz [12] and Hurdel et al. [13] using the finite element method, and by Panchang et al. [14] using the finite difference method. Panchang et al. [14] gave a detailed review of the numerical solution of the mild-slope equation. Compared with other numerical modeling approaches, the models based on the mild-slope equation are relatively fast and straightforward and allow estimation of irregular wave effects by a linear superposition of regular wave solutions.

On the other hand, the treatment of the boundary is very important for an effective numerical model. Berkhoff [1] pointed out that a coastal boundary condition for this elliptic partial differential equation contains two parameters, a reflection coefficient and a phase shift. The reflection coefficient represents a ratio of the amplitude of waves reflected away from a coastal boundary to the amplitude of waves approaching the coastal boundary. The reflection coefficient has a large effect on the wave field at locations near a coast. The phase shift occurs between approaching and reflected waves at the boundary. However, Isaacson [15] pointed out that the phase shift has only a small effect on the simulated wave field, and therefore it can be typically assumed to be zero. In addition, one of the factors affecting the treatment of boundaries is the direction of the approaching waves relative to the coastal boundary. Because the direction is unknown a priori, it is usually assumed to be zero for purposes of numerical calculation. However, Behrendt [16] demonstrated that the direction of the approaching waves has a considerable effect on the treatment of the boundary condition. The larger the direction angle, the greater the effect of undesired reflections from the boundary. Behrendt [16] also showed that of all the absorption–reflection conditions, the total absorption condition is the most affected by the direction of approaching waves. To take into account the effects of the angle on the simulated waves, Isaacson and Qu [17] suggested using an iterative method to deal with the unknown angle. Recently, Steward and Panchang [18] and Beltrami et al. [19] have presented detailed iterative methods. However, although Silva et al. [20] showed that the open boundary and partially reflecting boundary conditions can be more accurately modeled using a higher-order approximation for the boundary conditions, the linear boundary conditions are still used for simplicity to show the effects of approaching wave direction on the treatment of the boundaries.

To construct a finite element numerical model (FEM), the first step is to generate an appropriate grid system. For a fixed problem, the number of elements directly determines the calculation time and capacity of the model. To obtain a comparatively accurate and economical grid system, a self-adaptive preconditioning numerical model using a finite element method with triangular elements is developed in this paper. In the model, only the coarse elements which can represent the properties of the boundary and topography are given, and the elements can be refined according to the wave length which is related to the local water depth and the input wave frequency. In addition, the iterative method is used to evaluate the wave direction angle, and the effects of this angle on the simulated wave field are introduced into the self-adaptive finite element model. The model is verified by the simulation of wave propagation in some typical domains and by a comparison of the numerical results of multidirectional wave propagation into a breakwater gap with corresponding experimental data. Finally, the comparison of the numerically calculated wave distribution in a real harbor and that by the commercial software MIKE 21 developed by DHI Water and Environment is given to show the efficiency of the model for engineering practice.

2. Description of the self-adaptive FEM model

2.1. Governing equations

The model uses the well-known mild-slope equation for harmonic waves derived by Berkhoff [1]:

$$\nabla \cdot (F \cdot \nabla \phi) + \omega^2 G \phi \approx 0, \tag{1}$$

where