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A redundant repairable system with imperfect coverage and fuzzy parameters

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Abstract

This paper proposes a procedure to construct the membership functions of the system characteristics of a redundant repairable system with two primary units and one standby in which the coverage factor is the same for an operating unit failure as that for a standby unit failure. Times to failure and times to repair of the operating and standby units are assumed to follow fuzzified exponential distributions. The α-cut approach is used to extract from the fuzzy repairable system a family of conventional crisp intervals for the desired system characteristics, determined with a set of parametric nonlinear programs using their membership functions. A numerical example is solved successfully to illustrate the practicality of the proposed approach. Because the system characteristics are governed by the membership functions, more information is provided for use by management, and because the redundant system is extended to the fuzzy environment, general repairable systems are represented more accurately and the analytic results are more useful for designers and practitioners.

Keywords: Availability; Fuzzy sets; Imperfect coverage; Mathematical programming; Mean time to failure

1. Introduction

Redundancy plays an important role in improving the reliability (or availability) of engineering systems. Standby redundant repairable systems have been studied extensively in the past (Kumar and Agarwal [1], Birolini [2], Yearout et al. [3], and detailed bibliographies are found in Osaki and Nakagawa [4] and Srinivasan and Subramanian [5]). A number of authors have investigated the two-unit redundant systems under different assumptions (see Goel and Shrivastava [6], de Almeida and Campello de Souza [7], Gururajan and Srinivasan [8], Subramanian and Anantharaman [9], Rajamanickam and Chandrasekar [10], Billinton and Pan [11], Yadavalli et al. [12], and Seo et al. [13]). However, almost all research considers perfect coverage for failures. In contrast, imperfect coverage means that it is not successful detection, location, and recovery from a failure.

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In particular, it may be impossible to replace a failed unit with a spare and then recover from a failure. (see Trivedi [14]).

Among studies considering imperfect coverage, Pham [15] examined a model of a high voltage system consisting of a power supply and two transmitters with imperfect coverage in which the failure rate of fault coverage is constant. Pham's model was extended by Moustafa [16] to a K-out-of-N system with imperfect coverage. Recently, Trivedi [14] proposed a reliability model with three phases of failure handling: failure detection, location, and recovery for continued service. Reported research has largely been concerned with obtaining measures of system effectiveness. However, while these results can be useful in analysis and assist the decision process, there are very few studies that establish a decision model which directly determines an optimum maintenance strategy. de Almeida and Campello de Souza [7] constructed a decision model using a Bayesian approach and selected utility functions. Their approach motivates us to develop an alternative method to analyze repairable systems in which the uncertainty of the parameters is accounted for using a fuzzy approach. Repairable systems formulated in this way have broader applications for reliability engineers and management than conventional models.

In the literature described above, times to failure and times to repair are required to follow certain (known) probability distributions with fixed parameters. However, in many real-world applications, the distributions may only be characterized subjectively; that is, failure and repair patterns are frequently summarized with everyday language descriptions of central tendencies, such as "the mean failure rate is approximately 3 per day" or "the mean time between failures is approximately 8/24 days," rather than with complete probability distributions. The looseness with which the system measures are reported is revealing of the uncertainty concerning these distributions. And because times to failure and times to repair are therefore possibilistic rather than probabilistic, the reliability (or availability) problem becomes one of decision making in the context of risk (see Holloway [17]). To broaden applications of reliability and availability analysis in engineering (see Cleland and Kocaoglu [18]), general science, and management (Holloway [17]), this article extends it to fuzzy environments (see Zadeh [19]).

There are many studies on stochastic models with fuzzy environments in recent literature base. Only few among these studies focus on repairable systems with fuzzy parameter patterns using parametric nonlinear programming (see Ke et al. [20] and Huang et al. [21]). Different from other models, our model provides (i) a suitable estimation value form uncertain environments and (ii) a comparison discussion of using fuzzy theory and conventional method. This paper is organized as follows. Section 2 presents a detailed model description and introduces the reliability and availability characteristics of the repairable system. In Section 3, the repairable system is extended to the fuzzy environment and membership functions for system characteristics are briefly discussed. In Section 4, a mathematical programming approach is developed to derive the membership functions for the mean time to failure ($MTTF$) and availability. To demonstrate the validity of the proposed solution approach, a realistic numerical example is presented and solved in Section 5. Conclusions are drawn in Section 6.

2. Model description, reliability, and availability

We study a redundant repairable system with two identical operating units, which work independently and simultaneously, and one standby. Consider it may be impossible covered on the failure of an operating unit (or standby), even when replacing a failed unit with a standby. A detailed description of the repairable system is given as follows.

1. The standby unit may fail before it is put into full operation and is continuously monitored by a failure detection device. Both operating and standby units are repairable. An operating unit fails independent of the state of the other operating unit and has an exponential time-to-failure distribution with rate parameter λ. When an operating unit fails, it is immediately replaced by the standby if it is available. The standby fails independently of the state of the operating units and has an exponential time-to-failure distribution with rate parameter θ $(0 \leqslant \theta \leqslant \lambda)$.
2. If an operating unit fails, it is immediately detected, located, and replaced with coverage probability c with the standby if it is available. It is assumed that replacement is instantaneous. We also assume that the cov-