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A secant algorithm with line search filter method for nonlinear optimization

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ABSTRACT

Filter methods were initially designed for nonlinear programming problems by Fletcher and Leyffer. In this paper we propose a secant algorithm with line search filter method for nonlinear equality constrained optimization. The algorithm yields the global convergence under some reasonable conditions. By using the Lagrangian function value in the filter we establish that the proposed algorithm can overcome the Maratos effect without using second order correction step, so that fast local superlinear convergence to second order sufficient local solution is achieved. The primary numerical results are presented to confirm the robustness and efficiency of our approach.

1. Introduction

Consider the following optimization problem with nonlinear equality constraint (NLP)

$$\min \quad f(x) \tag{1.1a}$$

$$\text{subject to} \quad c(x) = 0, \tag{1.1b}$$

where $f(x):\mathbb{R}^n \to \mathbb{R}^1$ and $c(x):\mathbb{R}^n \to \mathbb{R}^m$ with $m < n$ are smooth. There are quite a few literatures proposing reduced Hessian algorithms in [1,2] and secant algorithms (two-step algorithms in [3]) to solve this problem. Compared with the widely reduced Hessian algorithms which refer to an orthonormal basis that might be difficult to find a continuous basis (see, for example, [4]), the secant algorithms have a main advantage which rests in the use of an orthogonal projection operator which is continuous.

For simplicity, we now only introduce Algorithm 3 of a family of secant algorithms presented by Fontecilla in [3]. Let $\|\cdot\|$ be the Eudidean norm on $\mathbb{R}^n$. For simplicity, we denote $f(x_k)$ by f_k; $\nabla f(x_k)$ by g_k; $\nabla^2_{xx} f(x_k)$ by $\nabla^2 f_k$; $c(x_k)$ by c_k; $A(x_k)$ by A_k; $\lambda(x_k)$ by λ_k; $\nabla\lambda(x_k)$ by $\nabla\lambda_k$ where $\nabla\lambda(x)$ is the Jacobian of $\lambda(x)$.

Given x_0, λ_0, B_0

For $k = 0$ Until convergence Do

$$\lambda_k = -\left(A_k^T A_k\right)^{-1} A_k^T g_k, \tag{1.2}$$

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$$B_k w_k = -\nabla_x \mathcal{L}(x_k, \lambda_k), \tag{1.3}$$
$$h_k = P(x_k, B_k) w_k, \tag{1.4}$$
$$v_k = -A_k^{\dagger} c_k, \tag{1.5}$$
$$s_k = h_k + v_k, \tag{1.6}$$
$$y_k = \nabla_x \mathcal{L}(x_k + h_k, \lambda_k) - \nabla_x \mathcal{L}(x_k, \lambda_k), \tag{1.7}$$
$$B_{k+1} = \text{DFP/BFGS}(h_k, y_k, B_k), \tag{1.8}$$
$$x_{k+1} = x_k + s_k. \tag{1.9}$$

In the above algorithm, $\mathcal{L}(x, \lambda)$ is the Lagrangian function defined by

$$\mathcal{L}(x, \lambda) = f(x) + \lambda^T c(x), \tag{1.10}$$

$P(x_k, B_k) = I - B^{-1} A_k \left(A_k^T B^{-1} A_k\right)^{-1} A_k^T$ is an oblique projection onto the null subspace $N\left(A_k^T\right)$, and $A_k^{\dagger} = A_k \left(A_k^T A_k\right)^{-1}$ is the Moore–Penrose inverse of A_k^T. In fact, $\lambda(x)$ can be found by solving the least squares problem

$$\min_{\lambda} \|A(x)\lambda + g(x)\|. \tag{1.11}$$

In the secant algorithm, h_k is used to update the matrices B_k by either the DFP or the BFGS secant update

$$B_{k+1}^{\text{DFP}} = B_k + \frac{(y_k - B_k h_k) y_k^T + y_k (y_k - B_k h_k)^T}{y_k^T h_k} - \frac{h_k^T (y_y - B_k h_k) y_k y_k^T}{\left(y_k^T h_k\right)^2}, \tag{1.12}$$

$$B_{k+1}^{\text{BFGS}} = B_k + \frac{y_k y_k^T}{y_k^T h_k} - \frac{B_k h_k (B_k h_k)^T}{h_k^T B_k h_k} \tag{1.13}$$

and v_k satisfies the linear constraint property

$$A_k^T v_k + c_k = 0.$$

Under the following assumption

$$\lim_{k \to \infty} \frac{\|P(x_k, B_k)(B_k - W_*) h_k\|}{\|h_k\|} = 0 \tag{1.14}$$

and some other reasonable conditions, the convergence rate is two-step Q-superlinear, i.e.,

$$\lim_{k \to \infty} \frac{\|x_{k+1} - x_*\|}{\|x_{k-1} - x_*\|} = 0. \tag{1.15}$$

Recently, Fletcher and Leyffer [5] proposed filter methods for nonlinear programming (NLP), offering an alternative to merit function method, as a tool to guarantee global convergence in algorithms for nonlinear programming (NLP). The underlying concept is that trial points are accepted if they improve the objective function or improve the constraint violation instead of a combination of those two measures defined by a merit function. Wächter and Biegler [6–8] gave a filter line search method for equality constrained nonlinear programming (NLP). In [9], they presented a primal–dual interior-point algorithm with a filter line search method for nonlinear programming and the practical results were encouraging. Gu and Zhu [10] presented a filter interior-point algorithm with projected Hessian updating for nonlinear optimization.

Stimulated by the progress in these aspects, we propose a secant algorithm with line search method for nonlinear equality constrained optimization. The main difference consists in using the Lagrange function value $f(x) + \lambda(x)^T c(x)$ instead of the objective function value in the filter together with an appropriate infeasibility measures. The algorithm yields the global convergence under some reasonable conditions. Further, we establish that the proposed algorithm can overcome the Maratos effect without using second order correction step which is important in theory. Moreover, since second derivatives are not required, our algorithm is sometimes more efficient than Newton's algorithm.

The paper is outlined as follows. In Section 2, we state the secant algorithm with line search filter method; the global convergence of the algorithm is proved in Section 3; the local two-step Q-superlinear convergence rate is established in Section 4; finally, we report some numerical experiments in Section 5.

2. Algorithm

Given a starting point x_0, the proposed line search method generates a sequence of improved estimates x_k of the solution for the NLP (1.1b). For this purpose in each iteration k a search direction s_k is computed by the secant algorithm at x_k. After a search direction s_k has been computed, a step size $\alpha_k \in (0, 1]$ is determined in order to obtain the next iterate

$$x_{k+1} = x_k + \alpha_k s_k. \tag{2.1}$$

In this paper we consider a backtracking line search procedure, where a decreasing sequence of step sized $\alpha_{k,l} \in (0, 1]$ $(l = 0, 1, 2, \ldots)$ is tried until some acceptance criterion is satisfied.