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Size-dependent pull-in phenomena in electrically actuated nanobeams incorporating surface energies

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ABSTRACT

A modified continuum model of electrically actuated nanobeams is presented by incorporating surface elasticity in this paper. The classical beam theory is adopted to model the bulk, while the bulk stresses along the surfaces of the bulk substrate are required to satisfy the surface balance equations of the continuum surface elasticity. On the basis of this modified beam theory the governing equation of an electrically actuated nanobeam is derived and a powerful technology, analog equation method (AEM) is applied to solve this complex problem. Beams made from two materials: aluminum and silicon are chosen as examples. The numerical results show that the pull-in phenomena in electrically actuated nanobeams are size-dependent. The effects of the surface energies on the static and dynamic responses, pull-in voltage and pull-in time are discussed.

1. Introduction

Microelectromechanical system (MEMS) can be defined as a system of small dimensions (less than one cubic centimeter) fulfilling a smart function. Recent years, with development of modern high-technique, the application of MEMS devices especially the electrically actuated MEMS devices which require few mechanical components and small voltage levels for actuation are continuously growing. The MEMS devices are widely used as capacitive accelerometer [1], capacitive sensor [2], switches [3] and so on. As mentioned above, compared to the traditional mechanical systems, the MEMS devices are usually small and for this large surface-to-volume ratio the integrated circuit (IC) technology in modern industry facilitates the fabrication of thousands of MEMS devices with increased reliability and reduced cost.

However, the actuated multiple physical field properties such as mechanical field, electric field, thermo field and so on in MEMS have made it a newly research frontier, as these physical field properties can bring out several instability responses of which the pull-in phenomenon is a typical one. The pull-in instability is a discontinuity related to the interplay of the elastic force and the electrostatic force, which is produced by the applied voltage. When the applied voltage increases beyond a critical value, called pull-in voltage, the elastic force can no longer resist the electrostatic force, thereby leading to collapse of the structure. A one dimensional (1D) lumped mode, which reduced a MEMS structure into a single rigid parallel-plate capacitor suspended above a fixed ground plane by an ideal linear spring, was used by Pamidighantam et al. [4] to show the pull-in phenomena of a micro-cantilever and a clamped–clamped microbeam where the effects of partial electric force axial stress, fringing fields and so on were all included in the final expression. By using the shooting method, Choi and Lovell [5] investigated the pull-in phenomena of a clamped–clamped microbeam where the midplane stretching effect in order to treat large deflections for clamped–clamped beams was taken into account. Based on the Galerkin procedure, Younis et al. [6]

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introduced a reduced-order computing model, and based on this model they showed the pull-in phenomena of a microbeam subjected to an electric actuation. Zhang and Zhao [7] used a new numerical method to give the pull-in voltage and the mechanical action of a RF MEMS switch, where the fringing field was considered. Considering the fringing field effect due to the finite beam width an improved electrostatic force of an electrically actuated narrow microbeam was set up by Batra et al. [8]. Recently, based on the viscoelastic theory, Fu and Zhang [9] have investigated the pull-in phenomenon and the nonlinear dynamic behavior of electrically actuated micro-cantilevers. In above articles, the mechanical models of MEMS are usually modeled by classical beam theory. However, when the dimension of MEMS comes to nano-scale, specially called as nanoelectromechanical system (NEMS), some experiments showed the pull-in phenomena of nanobeams are size-dependence [10] while the classical beam model used in above literatures can not explain such phenomena. Therefore, a modified beam model which can incorporate the size effect is desirable in NEMS.

Fortunately, it is found that the continuum model by incorporating surface elasticity can predict the same accurate elastic response of nanobeams as the case of atomistic modeling if the proper surface constitutive constants are used [11]. The surface energy was considered by Gurtin and Murdoch [12,13] to develop the surface elasticity theory for isotropic materials based on some rational principles of mechanics. In their model, the surface layer of a solid is treated as a membrane with negligible thickness and perfectly bonded to the underlying bulk. In addition, a set of constitutive equations and the generalized Young–Laplace equation are applied to the surface. The surface elasticity theory by Gurtin and Murdoch [12,13] have received increasing interests in more recent researches in studying some mechanical problems in structural elements with nano-scale dimensions, e.g. He et al. [14] proposed a rigorous continuum surface elasticity model and successfully analyzed the size-dependent deformation of nanofilms. Lim and He [15] developed a model based on the Gurtin and Murdoch theory to analyze the deformation of nanofilms under bending. Lu et al. [16] generalized the thin plate model to include the normal stresses in the bulk, and presented a modified theory for thin and thick plates. However, to our best knowledge, the application of Gurtin and Murdoch theory to the analysis of the size-dependent pull-in phenomena in MEMS/NEMS structures has not been reported.

In this paper, a modified continuum model of electrically actuated nanobeams by incorporating surface elasticity is presented. The surface layer and bulk of the beam are assumed elastically isotropic. Gurtin and Murdoch's theory of surface elasticity is applied to describe the surface layer, while the Euler–Bernoulli hypothesis is used to model the bulk deformation kinematics. The influence of the geometrical nonlinear is also considered. The complex mathematical problem is solved by the analog equation method (AEM) and the effects of the surface energies on the static and dynamic responses, pull-in voltage and pull-in time are discussed.

2. Problem formulation

Consider an arbitrary cross-section (symmetric about the z-axis) of a nanobeam as shown in Fig. 1. It is assumed that the response of the beam is governed by the continuum theory proposed by Gurtin and Murdoch [12,13]. Unlike the classical case, a beam based on the Gurtin–Murdoch continuum model is considered to have an elastic surface (mathematically zero thickness) perfectly bonded to its bulk material. The elastic surface has distinct material properties and accounts for the surface energy effects. As in the case of classical beam theory, the stress state of the bulk material of the beam is plane stress with the non-zero stresses σ_{xx}, σ_{xz} and σ_{zz}. The relevant bulk strains are ε_{xx}, ε_{xz} and ε_{zz}, respectively. The elastic surface has non-zero membrane stress τ_{xx} and τ_{nx} where n denotes the outward normal of the surface. A free-body diagram of an incremental beam element of length dx is shown in Fig. 2 and it is assumed that an element of beam is displaced equal to w in the z-axis. As a result of the interaction between the surface layer and bulk material, the contact tractions $T_i = \sigma_{ij}n_j$ exist on the contact surface between the bulk material and surface layer. In view of the plane stress state, only T_x and T_z are non-zero. Assume the bending moment, the axial force and the shear force of a bulk material cross-section be M, N and Q, respectively. Then according to the bending moment and vertical force equilibrium equations of the element: $\sum F_z = 0$ and $\sum M_{0'} = 0$, the following equations can be obtained

$$\frac{\partial Q}{\partial x} + \int_s T_z\, ds + q(x,t) - \int_A \rho \frac{\partial^2 w}{\partial t^2}\, dA = 0, \tag{1}$$

$$\frac{\partial M}{\partial x} - \int_s T_x z\, ds + Q - N\frac{\partial w}{\partial x} = 0, \tag{2}$$

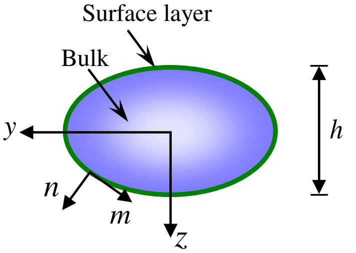

Fig. 1. An arbitrary cross-section of the nanobeam.