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Accuracy of the two-iteration spectral method for phase change problems

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Abstract

The accuracy of the solution of phase change problems using a spectral method is studied. Two iterations in the expansion are used to obtain the interface location of a solidification problem in semi-infinite domain. Asymptotic expansion of the current approach is compared to the existing analytical solution of the problem, and the validity of the expansion is studied. The results indicate the accuracy of a numerical application of the current approach to finite and semi-infinite geometries.

Keywords: Moving boundary problem; Spectral method; Phase change

1. Introduction

Moving boundary problems have been of interest in numerous fields. In addition to phase change, moving boundary problems are encountered in diverse phenomena, such as growth of crystals. Due to the presence of the moving boundary, analytical solutions are rare, and so far have been restricted to semi-infinite geometries [1–3]. Various approximate methods have been used for solving moving boundary problems. Many books [4–6], and survey articles [7] give

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accounts of the methods used. The spectral approach was used in the solution of solidification problems in finite domains with constant and time dependent boundary temperatures [8,9]. After fitting a Fourier series for the spatial variation of temperatures, the resulting differential equation for the interface location was solved numerically and the temperature distributions were computed. Two iterations were used in the approximation. The results of the numerical approach in [8] were tested via comparison with existing approximate and analytical solutions and found to be valid.

In the current study the accuracy and limitations of the two-iteration scheme is assessed. An analytical series solution of the solidification problem in a semi-infinite medium is obtained using the approach of [8,9]. Unlike [8,9], the iterative integration of the resulting integro-differential equation is obtained analytically, and the result enabled a direct comparison with the existing analytical solution.

2. Formulation

The problem in question is the solidification of a finite region, initially at a constant temperature T_i, which is higher than the melting temperature T_m. The solution for the semi-infinite region is obtained by extending the width of the region to infinity. The boundary at $x = 0$ is suddenly subjected to a temperature T_w that is less than the fusion temperature, a phase change interface is formed and starts to advance into the liquid region. The energy equations for the two phases are given as,

$$\alpha_1 \frac{\partial^2 T_1}{\partial x^2} = \frac{\partial T_1}{\partial t}, \quad 0 \leqslant x \leqslant s, \tag{1}$$

$$\alpha_2 \frac{\partial^2 T_2}{\partial x^2} = \frac{\partial T_2}{\partial t}, \quad s \leqslant x \leqslant l, \tag{2}$$

where the subscripts 1 and 2 represent the solid and liquid phases, respectively, and α_i is the thermal diffusivity. The interface location is denoted by s and the temporal domain is $0 < t < \infty$. The initial and boundary conditions are,

$$T_1(0,t) = T_w, \quad T_2(x,0) = T_i, \quad \frac{\partial T_2}{\partial x}(l,t) = 0, \quad s(0) = 0 \tag{3}$$

and an energy balance at the phase change interface gives,

$$k_1 \frac{\partial T_1}{\partial x} - k_2 \frac{\partial T_2}{\partial x} = \rho L \frac{ds}{dt}, \tag{4}$$

where the constants k, ρ, L represent thermal conductivity, density and the latent heat of fusion, respectively. The interface remains at the melting temperature,

$$T_1(s,t) = T_2(s,t) = T_m. \tag{5}$$

In addition to the presence of a moving boundary, where the interface condition is satisfied, the interface energy balance couples the energy equations in the two phases, resulting in a complex problem to be analyzed. Introduction of the dimensionless temperature θ,