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Applied Mathematical Modelling 30 (2006) 1525–1534
APPLIED MATHEMATICAL MODELLING
www.elsevier.com/locate/apm

A new approach to universal approximation of fuzzy functions on a discrete set of points

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Received 1 January 2003; received in revised form 1 July 2005; accepted 14 July 2005
Available online 12 September 2005

Abstract

One of the interesting, important and attractive problems in applied mathematics is approximation of functions in a given space. In this paper the problem is considered for fuzzy data and fuzzy functions using the defuzzification function of Fortemps and Roubens. Approximation of a fuzzy function on some given points $(x_i, \tilde{f}_i)$ for $i = 1, 2, \ldots, m$ is considered by some researchers as interpolation problem. But in interpolation problem we find a polynomial of degree at most $n = m - 1$ where m is the number of points. But when we have lots of points (m is very large) it is not good or even possible to find such polynomials. In this case we want to find a polynomial of arbitrary degree which is an approximation to original function. One of the works has done is regression by some researchers and we introduced a different method. In this case we have m points but we want to find a polynomial of degree at most $n < m$ but not $n = m - 1$ necessarily. We introduce a fuzzy polynomial approximation as universal approximation of a fuzzy function on a discrete set of points and we present a method to compute it. We show that this approximation can be non-unique, however we choose one of them with the smallest amount of fuzziness.

Keywords: Fuzzy approximation; Fuzzy linear programming

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doi:10.1016/j.apm.2005.07.004

1. Introduction

The interpolation problem of fuzzy data first was introduced by Zadeh [1]. Lowen [2] presented the Lagrange interpolation problem. Kaleva [3] introduced some properties of Lagrange and cubic spline interpolation. Properties of natural splines and complete splines of odd degrees are introduced in [4,5]. In interpolation problem there are given $n+1$ distinct points in $\mathbb{R}$ and for each points there is given a fuzzy value in $\mathbb{R}$, we find a fuzzy polynomial of degree at most n which coincides, on these points with given fuzzy values.

In this work we first introduce the basic concepts of fuzzy numbers, and a kind of fuzzy polynomial $\widetilde{P}_n(x)$ of degree at most n where $x \in \mathbb{R}$. Here we are given m distinct points in $\mathbb{R}$, and for each points we have a fuzzy value in $\mathbb{R}$. We want to find a fuzzy polynomial of degree at most n which n is not necessarily equal to $m-1$ (in this paper we consider $m > n$) and sometimes m is very large. The authors already introduced a method for solving such problems in [6].

In Section 3 we introduce universal approximation of a fuzzy function and a method for computing it with respect to a specified ranking and using linear programming. But does the universal approximation of a fuzzy function always exist? And is it unique? We will answer these questions in Section 4. At last some examples are given in Section 5. Throughout this paper we use standard difference for fuzzy numbers.

2. Basic concepts

Let $F(\mathbb{R})$ be the set of all real fuzzy numbers (which are normal, upper semi-continuous, fuzzy convex and compactly supported), and $TF(\mathbb{R})$ be the set of all triangular fuzzy numbers, which is

$$\mu_{\tilde{v}}(x) = \begin{cases} 1 - \dfrac{v_c - x}{v_l}, & (v_c - v_l) \leqslant x \leqslant v_c, \\ 1 + \dfrac{v_c - x}{v_r}, & v_c \leqslant x \leqslant (v_c + v_r), \\ 0, & \text{otherwise} \end{cases}$$

and denoted by $\tilde{v} = (v_c, v_l, v_r)$.

In [7], *parametric form* of a fuzzy number has been introduced and presented by $\tilde{v} = (\underline{v}(r), \overline{v}(r))$, where functions $\underline{v}(r)$ and $\overline{v}(r)$; $0 \leqslant r \leqslant 1$ satisfying the following requirements:

1. $\underline{v}(r)$ is monotonically increasing left continuous function;
2. $\overline{v}(r)$ is monotonically decreasing left continuous function;
3. $\underline{v}(r) \leqslant \overline{v}(r)$, $0 \leqslant r \leqslant 1$;
4. $\overline{v}(r) = \underline{v}(r) = 0$ for $r < 0$ or $r > 1$.

A crisp number α, can be simply represented by $\overline{v}(r) = \underline{v}(r) = \alpha$, $0 \leqslant r \leqslant 1$.

Let $\tilde{v} = (\underline{v}(r), \overline{v}(r))$, $\tilde{u} = (\underline{u}(r), \overline{u}(r)) \in F(\mathbb{R})$. Some results of applying fuzzy arithmetic on fuzzy numbers $\tilde{v}$ and $\tilde{u}$ are as follows: