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Parametric modelling and numerical simulation of natural-convective transport of radon-222 from a phosphogypsum stack into open air

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Abstract

An existing finite-volume computational simulator for heat and mass transfer in media fully or partially filled with porous material has been adapted to predict radon-222 exhalation rates. For validation purposes, this paper numerically examines the extent of natural-convective effects on radon-222 steady-state transfer from a phosphogypsum stack into the surrounding atmosphere. The stack is approximated by a dry rectangular porous matrix having uniform porosity and isotropic permeability whereas the supposedly laminar buoyancy-driven air flow is modelled following Darcy–Brinkman–Boussinesq approach. Differential governing equations are cast in dimensionless form in order to encompass simultaneous effects from physical factors involved. Dimensionless groups related to decay and emanation processes are put forward apart from usual controlling parameters such as Prandtl, Schmidt, Darcy and Grashof numbers. Results are reported for $10^6 \leqslant Gr \leqslant 10^8$ and $10^{-13} \leqslant Da \leqslant 10^{-7}$. Natural-convective effects on typical low-permeability phosphogypsum stacks proved to be of minor importance, as radon-222 transfer becomes diffusive dominant as expected.

Keywords: Modeling; Simulation; Porous media; Radon exhalation; Phosphogypsum

1. Introduction

Demands for phosphate fertilizers have by-produced tons of phosphogypsum ($CaSO_4 \cdot 2H_2O$) worldwide for years [1]. Although potential commercial exploitations have been sought, such waste has been simply piled up [2]. Exposed phosphogypsum raises environmental concerns related to ^{222}Rn exhalation resulting from the alpha-decay of ^{226}Ra, which is a radionuclide commonly found in phosphogypsum as an impurity.

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Nomenclature

A	stack aspect ratio (dimensionless, based on half-width)
c	activity concentration (Bq m^{-3} for air-borne radon; Bq kg^{-1} for radium)
D	radon (mass) diffusivity in air ($m^2\ s^{-1}$)
Da	Darcy number (dimensionless)
f	emanation coefficient (dimensionless)
g	gravity acceleration ($m\ s^{-2}$)
$\widetilde{G}$	activity generation rate per unit of REV bulk volume (Bq $m^{-3}\ s^{-1}$)
Gr	Grashof number (dimensionless)
H	stack height (m)
j	activity flux (Bq $m^{-2}\ s^{-1}$)
J	dimensionless activity flux
K	stack permeability (m^2)
L	stack half-width (m)
M	emanation-to-decay ratio (dimensionless)
n	parameter to indicate flow inside or outside porous matrix (dimensionless)
Nu	Nusselt number (dimensionless)
p	pressure (Pa)
P	dimensionless pressure
Pr	Prandtl number (dimensionless)
R	decay-to-diffusion ratio (dimensionless)
S	emanation-to-diffusion ratio (dimensionless)
Sc	Schmidt number (dimensionless)
Sh	Sherwood number (dimensionless)
T	temperature (K)
u, v	Cartesian velocity components ($m\ s^{-1}$)
U, V	dimensionless Cartesian velocity components
x, y	Cartesian coordinates (m)
X, Y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity ($m^2\ s^{-1}$)
β	thermal expansion coefficient (K^{-1})
γ	activity transfer coefficient ($m\ s^{-1}$)
Γ	bulk(effective)-to-fluid kinematic viscosity ratio (dimensionless)
ε	porosity (dimensionless)
θ	dimensionless temperature
λ	radon-222 decay constant ($2.098 \times 10^{-6}\ s^{-1}$)
Λ	bulk-to-fluid thermal diffusivity ratio (dimensionless)
υ	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
ϕ	dimensionless activity concentration
Ψ	bulk-to-fluid (open air) radon diffusivity ratio (dimensionless)

Subscripts and superscripts

a	interstitial air
c	convective flux
d	diffusive flux
pc	partition-corrected