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Finite element analysis of thermoelastic field in a rotating FGM circular disk

A.M. Afsar *, J. Go

Department of Mechanical Engineering, Changwon National University, Changwon, Gyeongnam 641-773, South Korea

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ABSTRACT

This study focuses on the finite element analysis of thermoelastic field in a thin circular functionally graded material (FGM) disk subjected to a thermal load and an inertia force due to rotation of the disk. Due to symmetry, the FGM disk is assumed to have exponential variation of material properties in radial direction only. As a result of nonuniform coefficient of thermal expansion (CTE) and nonuniform temperature distribution, the disk experiences an incompatible eigenstrain which is taken into account. Based on the two dimensional thermoelastic theories, the axisymmetric problem is formulated in terms of a second order ordinary differential equation which is solved by finite element method. Some numerical results of thermoelastic field are presented and discussed for an Al_2O_3/Al FGM disk. The analysis of the numerical results reveals that the thermoelastic field in an FGM disk is significantly influenced by temperature distribution profile, radial thickness of the disk, angular speed of the disk, and the inner and outer surface temperature difference, and can be controlled by controlling these parameters.

1. Introduction

Functionally graded materials (FGMs) have continuously varying material composition from one surface to another surface which makes these materials nonhomogeneous in terms of both the material properties and microstructures. Usually, a ceramic is used at one surface to resist severe environmental effect, such as high temperature, wear, and corrosion; and a metal is used at the other surface to ensure higher toughness and thermal conductivity. The original purpose of these materials was to develop thermal barrier coatings for propulsion system of spaceplanes to resist high temperature and ensure high thermal conductivity at the same time [1]. From a mechanics viewpoint, the main advantages of material property grading appear to be improved bonding strength, toughness, wear and corrosion resistance, and reduced residual and thermal stresses. Therefore, now-a-days, an FGM has been a promising candidate for many engineering applications where a high temperature gradient field is the main concern.

Because of the outstanding advantages of FGMs over conventional composites and monolithic materials, these materials have been extensively studied for potential applications as structural elements, such as FGM beams, plates, shells, and cylinders. Yongdong et al. [2] and Zhong and Yu [3] have considered FGM beams of rectangular cross section for the analysis of stress due to mechanical loads. Thermal load was considered by Xiang and Yang [4] for the analysis of free and forced vibration of a laminated functionally graded Timoshenko beam of variable thickness. A Timoshenko beam of FGMs was also considered to investigate the post buckling behavior in response to the thermal load [5]. FGM beams have been considered not only for stress, vibration, and buckling analyses but also for the analysis of fracture behavior [6].

Buckling characteristics of rectangular FGM plates were analyzed by Chen and Liew [7] for nonlinearly distributed in-plane edge loads and by Feldman and Aboudi [8] for uniaxial load. Assuming the material properties to be varied in

* Corresponding author. Tel.: +82 010 2892 3397; fax: +82 55 267 5142.
E-mail address: mdafsarali1967@yahoo.com (A.M. Afsar).

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the thickness direction according to the sigmoid function, geometrically nonlinear analysis was carried out for FGM plates and shells by Kim et al. [9]. The solutions of axisymmetric bending and buckling of an FGM circular plate based on third order plate theory and classical plate theory have been discussed by Ma and Wang [10]. In our previous study [11], an FGM rectangular plate under mechanical load was considered with a view to examining the effect of nonhomogeneous parameter on the elastic field. Chung and Chang [12] also considered an FGM rectangular plate to study the mechanical behavior of the plate subjected to thermal loadings.

FGM circular cylinder and hollow sphere were considered by Obata and Noda [13] to carry out the analysis of thermal stresses. A similar study was carried out by Liew et al. [14] to investigate the thermal stress behavior of an FGM circular cylinder. As the FGMs have continuously varying coefficient of thermal expansion, an eigenstrain [15] is inherently developed in the materials when they undergo a change in temperature. The incompatibility of this eigenstrain causes an eigenstress or thermal residual stress to develop in these materials. Thus, the incompatible eigenstrain is an important issue to be considered in characterizing the FGM bodies. By taking into account the effect of incompatible eigenstrain, Afsar and Sekine [16], Afsar and Anisuzzaman [17], and Afsar et al. [18] analyzed the brittle fracture characteristics of thick-walled FGM cylinders.

In addition to the above static analyses of FGM bodies, a few studies on dynamic behavior of FGM bodies are also available in the literature. Among other geometries of FGM bodies, a rotating FGM hollow shaft with fixed ends was chosen by Eraslan and Akis [19] for estimating the elastoplastic response of the shaft and a rotating FGM sandwich solid disk with material gradient in the thickness direction was considered by Zenkour [20] for the analysis of stress and displacement.

It is evident from the above discussion that extensive studies on FGMs have been carried out to explore their potential applications mainly as structural elements like beams, plates, shells, and cylinders. The functional applications of these materials, especially in the field of cutting and grinding tools, appear to be inadequate in the literatures. Cho and Park [21] have investigated the thermoelastic characteristics of functionally graded lathe cutting tools composed of Cr–Mo steel shank and ceramic tip with a view to exploring its thermomechanical superiority. They found that the thermomechanical stress concentration was significantly relaxed by adding an FGM layer between the steel shank and the ceramic tip and thus justified the potential of FGMs for high performance metal cutting tools. The authors of the present paper are motivated to investigate the potential applications of FGMs in a circular disk type cutter or a grinding disk for better performance. This type of elements experiences a higher temperature at the cutting edge or grinding surface, when it rotates and comes in contact with an object, and a lower temperature at the central region of the disk. Further, the cutting edge or grinding surface of the tool should be harder and have high wear resistance while the central part should have a higher toughness. A uniform material or bi-materials cannot meet these requirements because a high thermal stress is developed in the case of a uniform or a bi-material that causes failure. On the other hand, these requirements of a disk can be easily met up by designing the disk with FGMs.

However, designing of an FGM cutter or a grinding disk requires understanding and quantifying the thermoelastic field in these elements. In an attempt to achieve this goal, this study concentrates on the analysis of thermoelastic field in a rotating FGM circular disk. Along with the thermal load and inertia force due to rotation of the disk, the study takes into account the effect of the incompatible eigenstrain developed in the disk due to nonuniform CTE. Here, it is worthy to mention that although such a disk type cutter or grinder experiences a contact load when it comes in contact with a work piece, it is not considered in the present study. The solution of contact problem of an FGM disk can be considered in another study and superposed with the solution of the present thermoelastic problem to obtain the resultant characteristics. The range of variation of the Poisson's ratio is small and, therefore, has an insignificant effect on the thermoelastic characteristics of the disk. Thus, the Poisson's ratio is assumed to be constant [10,22] while other material properties, such as Young's modulus, CTE, and density of the FGM disk, are assumed to vary exponentially only in the radial direction due to symmetry with respect to the axis of the disk. Based on the two dimensional thermoelastic theories, the problem is formulated in terms of a second order differential equation, which is solved by finite element method. Some numerical results of the thermoelastic field are presented for an Al_2O_3/Al FGM disk and the effects of temperature distribution profile, radial thickness of the disk, angular speed of the disk, and the inner surface and outer surface temperature difference are analyzed in details.

2. Mathematical model of the problem

Shown in Fig. 1 is a rotating FGM circular disk with a concentric circular hole. The origin of the polar coordinate system $r-\theta$ is assumed to be located at the center of the disk and hole. Further, the FGM disk is considered to be made of two distinct material phases A and B which are, respectively, represented by the dark and white colors as shown in the figure. The distribution of each material continuously varies along the radial direction only which makes the problem axisymmetric. The radii of the hole and outer surface of the disk are designated by a and b, respectively. Further, the angular velocity of the disk is denoted by ω. As the material distribution is the function of r only, all the properties of the FGM disk are also functions of r only. The Young's modulus E, CTE α, and the density ρ of the FGM disk are assumed to vary exponentially as

$$E = E_0 e^{\beta r}, \tag{1a}$$

$$\alpha = \alpha_0 e^{\gamma r}, \tag{1b}$$

$$\rho = \rho_0 e^{\mu r}. \tag{1c}$$