Applied Mathematical Modelling 35 (2011) 3769–3776

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Using Zionts–Wallenius method to improve estimate of value efficiency in DEA

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ARTICLE INFO

Article history:
Received 10 January 2010
Received in revised form 29 January 2011
Accepted 2 February 2011
Available online 3 March 2011

Keywords:
Data envelopment analysis
Value efficiency
Efficiency analysis
Multiple objective linear programming
Value function

ABSTRACT

Halme et al. [M. Halme, T. Joro, P. Korhonen, S. Salo, J. Wallenius, A value efficiency approach to incorporating preference information in data envelopment analysis, Management Science 45 (1) (1999) 103-115] proposed value efficiency analysis as an approach to incorporate preference information in Data Envelopment Analysis (DEA). In this paper, we develop some related concepts, and present a refinement to Halme et al.'s approach to measure value efficiency scores more precisely. For do this, we will introduce an MOLP model which its objective functions are input/output variables subject to the defining constraints of Production Possibility Set (PPS) of DEA models. Then by using the so-called Zionts–Wallenius method, we aid the Decision Maker (DM) in searching for the Most Preferred Solution (MPS) and generating input/output weights as the DM's underlying value structure about objective functions. Finally, value efficiency scores are calculated by comparing the inefficient units to units having the same value as the MPS.

1. Introduction

Data envelopment analysis (DEA), Charnes et al. [1], is originally value-free and it was considered an advantage of DEA that no preference information is needed. However, because DEA parameters are influenced by the structure of the production process under analysis, the weights obtained from a standard DEA analysis are not a suitable measure of the preferences of a given decision maker (DM).

Several attempts have been made to incorporate preference information in DEA. Weights restrictions, Dyson and Thanassoulis [2] and Wong and Beasley [3], are the most straightforward way to do this. Outside of weights restrictions, several researchers were concerned with the determination of efficiency scores and targets based on preference structures and linkage between DEA and multi-criteria decision making (MCDM); see [4–6], and others.

Another way of utilizing the preference information is value efficiency analysis, Halme et al. [7]. The basic idea of Halme et al.'s approach was to measure efficiency of each decision making unit (DMU) as a distance to an approximated indifference contour of a DM's value function.

Here, we present a refinement to Halme et al.'s approach by introducing an MOLP model which its objective functions are input/output variables subject to the defining constraints of production possibility set (PPS) in standard DEA models. Then by using the so-called Zionts–Wallenius method, we aid the DM in searching for the most preferred solution (MPS). A special feature of using this method is generating objectives weights as the DM's underlying value structure about objectives. In other words, considering the structure of objective functions of the proposed MOLP model, these weights can be considered as the preference weights produced based on the DM's underlying value structure about inputs/outputs. Using these input/output weights we can introduce tangent hyperplane which approximate the indifference contour of the unknown value

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doi:10.1016/j.apm.2011.02.027

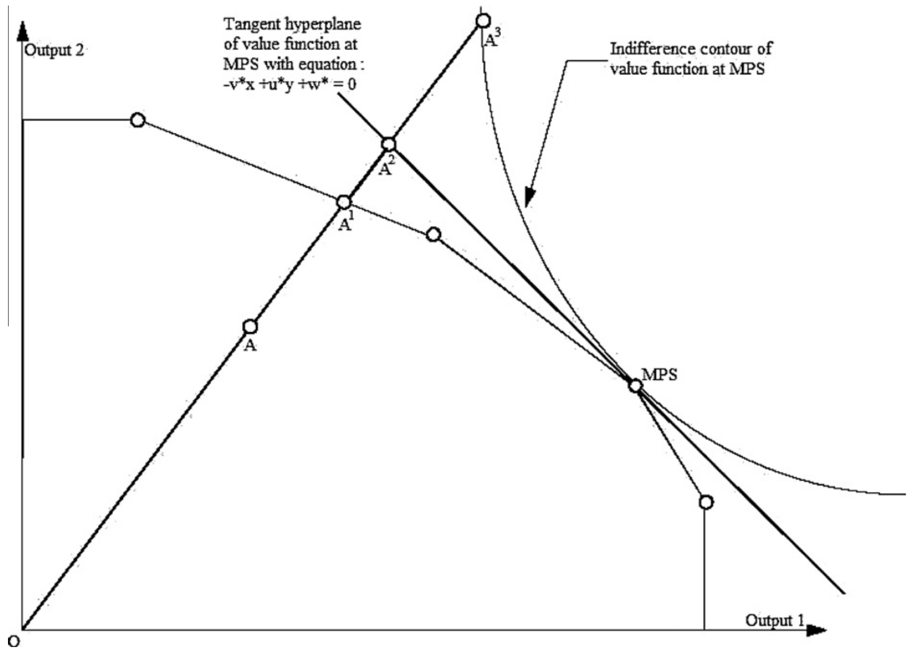

Fig. 1. Illustration of value efficiency analysis.

function at MPS. Finally, value efficiency scores are calculated for each DMU, without solving any LP problems, comparing the inefficient units to units having the same value as the MPS.

The paper is organized as follows. In Section 2 we present a brief discussion about value efficiency concept and Zionts–Wallenius method in solving MOLP problems. The mathematical foundation of our method to produce a more precise measure of the value efficiency scores is discussed in Section 3. Numerical example is presented in Section 4 and finally, Section 5 draws the conclusive remarks.

2. Literature review

Value efficiency is an efficiency concept, as an extension of the concept of efficiency in DEA, which takes into account the DM's preferences. This method is based on the assumption that the DM compares alternatives using a value function. The value function is assumed to be pseudoconcave, strictly increasing in outputs, strictly decreasing in inputs and reach its maximum at MPS on the efficient frontier. However, to be able to calculate the true value efficiency scores, one must know the value function, where in practice, unfortunately, this assumption is not realistic and we have to approximate this function.

The basic idea of value efficiency analysis is illustrated in Fig. 1. We assume that the DMUs produce two outputs and all consume the same amount of one input. For evaluating point A, we would like to evaluate the ratio $\frac{OA^3}{OA}$, but because the value function is unknown, if we could approximate the indifference contour by a tangent hyperplane, then we could use the ratio $\frac{OA^2}{OA}$. For more theoretical and computational considerations to this concept see [7–10].

Suppose that (x^*,y^*) be the MPS. In fact, the true value efficiency score tells how much a $\mathrm{DMU}_o = (x_o, y_o)$ has to improve its current input/output values in the direction (W_x, W_y) in order to be as preferred as the MPS. But because the value function is unknown, we want to approximate the value θ such that $(x_o, y_o) + \theta(W_x, W_y) = (\bar{x}, \bar{y})$, where $(\bar{x}, \bar{y})$ is the projected point of DMU_o on the tangent hyperplane of value function at MPS. Due to the equation of this hyperplane, we have: $-v^*(x_o + \theta W_x) + u^*(y_o + \theta W_y) + w^* = 0$. In other words, $\theta = -\frac{-v^* x_o + u^* y_o + w^*}{-v^* W_x + u^* W_y}$.

It is evident that this value is positive if and only if the DMU is value inefficient. Furthermore, for output oriented direction $(W_x, W_y) = (0, y_o)$ we have: $\theta = \frac{v^* x_o - w^*}{u^* y_o} - 1 = \frac{OA^2}{OA} - 1 = \frac{AA^2}{OA}$.

2.1. Value efficiency approaches

Halme et al.'s procedure began by aiding the DM in searching for the MPS, and approximating the indifference contour of the value function at this point with the region containing all vectors surely less than or equally preferred to the MPS, which can be characterized by the tangent hyperplanes of all possible pseudoconcave value functions obtaining their maximum at the MPS. Those hyperplanes define a new efficiency frontier and efficiency is then defined in relation to this frontier using a standard DEA technique. However, because they use an approximation and not the true indifference contour of the value