Available online at www.sciencedirect.com

ScienceDirect

Applied Mathematical Modelling 32 (2008) 1696–1710

APPLIED MATHEMATICAL MODELLING

www.elsevier.com/locate/apm

Modeling and control of surface gravity waves in a model of a copper converter

Eduardo Godoy [c], Axel Osses [a,c], Jaime H. Ortega [b,c], Alvaro Valencia [d,*]

[a] *Departamento de Ingeniería Matemática, Universidad de Chile, Casilla 170/3, Correo 3, Santiago, Chile*
[b] *Departamento de Ciencias Básicas, Fac. de Ciencias, Universidad del Bío-Bío, Campus Fernando-May, Casilla 447, Chillán, Chile*
[c] *Centro de Modelamiento Matemático, UMI 2807 CNRS U. de Chile, Casilla 170-3, Correo 3, Santiago, Chile*
[d] *Departamento de Ingeniería Mecánica, Universidad de Chile, Casilla 2777, Santiago, Chile*

Received 1 August 2005; received in revised form 1 April 2007; accepted 4 June 2007
Available online 16 June 2007

Abstract

Undesirable splashing appears in copper converters when air is injected into the molten matte to trigger the conversion process. We consider here a cylindrical container horizontally placed and containing water, where gravity waves on the liquid surface are generated due to water injection through a lateral submerged nozzle. The fluid dynamics in a transversal section of the converter is modeled by a 2-D inviscid potential flow involving a gravity wave equation with local damping on the liquid surface. Once the model is established, using a finite element method, the corresponding natural frequencies and normal modes are numerically computed in the absence of injection, and the solution of the system with injection is obtained using the spectrum. If a finite number of modes is considered, this approximation leads to a system of ordinary differential equations where the input is represented by the fluid injection. The dynamics is simulated as perturbations around a constant fluid injection solution, which is the desired operating state of the system, considering that the conversion process does not have to be stopped or seriously affected by the control. The solution is naturally unstable without control and the resulting increase of amplitude of the surface waves are assimilable to the splashing inside the converter. We show numerically that a variable flow around the operating injection is able to sensibly reduce these waves. This control is obtained by a LQG feedback law by measuring the elevation of the free surface at the point corresponding to the opposite extreme to where the nozzle injection is placed.

Keywords: Copper converter; Gravity waves; Potential flow; Kelvin–Voigt damping; LQG control

1. Introduction

Copper converters carry out the copper concentrate fusion and conversion process. The injection of air jets into the molten matte bath through submerged tuyeres plays a fundamental role because the interaction

* Corresponding author.
E-mail addresses: edgodoy@dim.uchile.cl (E. Godoy), axosses@dim.uchile.cl (A. Osses), jortega@dim.uchile.cl (J.H. Ortega), alvalenc@ing.uchile.cl (A. Valencia).

0307-904X/$ - see front matter
doi:10.1016/j.apm.2007.06.015

between air and matte produces the necessary chemical reactions required for the conversion process [1]. Nevertheless, the air jets also cause undesirable effects, such as excessive splashing and bath agitation. Over time, this splashing spoils the internal walls, shortening the converter's useful life. Therefore, to find a way to reduce this splashing could become an important technological progress.

In [2], Valencia et al. have numerically studied splashing by air injection through a tuyere into water within a cylindrical vessel. They have implemented the numerical simulations using the commercial software Fluent, involving a complex 3-D biphasic turbulent flow slice model. The standing waves on the liquid surface have been also calculated by assuming potential flow and gravity waves. They model the gas jet trajectory into the liquid following the theoretical equation of Temelis et al. [3].

Complementing this previous study, the present study proposes a simplified mathematical model of gravity waves in a copper converter and their control by variable fluid injection. Similarly, as done in [2] for standing waves, the fluid dynamics is described as a 2-D inviscid potential flow and a gravity wave equation posed on the liquid free surface. The jet effect is simplified assuming that water instead of gas is injected through a submerged horizontal nozzle. A second nozzle is placed on the converter bottom, which extracts the injected water, in such a way that the fluid volume inside the converter is constant. In order to better describe the physical phenomena, a local damping term is added to the wave equation on the free surface, which makes the model energy-dissipative. A general work on this matter is [4], where Burns and King have presented a note on the modeling of second-order systems subjected to different kinds of local damping. In [5], Liu and Rao have mathematically analyzed the exponential stability of the wave equation with Kelvin–Voigt local damping.

The numerical computation of the natural frequencies and normal modes in the 2-D domain allows the transformation of the time-dependent part of the damped gravity wave equation on the free surface into a linear first-order ODE's system. This system can be used to implement a gravity wave control method based on the time-variable fluid injection rate. It is important to note that this injection rate can not be chosen arbitrarily since it is necessary to control without considerably interfering in the processes that take place in the converter. Therefore, we assume that only small variations of this injection rate around a reference value (characterized by some given Froude number) are allowed. In order to find an appropriate time-variable injection, we consider the linear quadratic gaussian LQG control (cf. [6]) taking into account the reference injection constraint, the errors of the model, the observations, and a minimal norm control criteria.

Let us mention other previous studies for controlling gravity waves and related to this work. In [7], the author has studied the control and stabilization of a perfect fluid in a channel by means of wave generators, whereas in [8], he has solved a similar problem to the one studied here that involves a liquid inside a rectangular container during transportation. In both cases, the liquid elevation at the extremes of the container represents the observation or output. We consider here the same type of observation, more precisely, the surface elevation measured at the opposite extreme where the injection nozzle is placed.

2. Problem statement

2.1. Mathematical model

Let us consider a transversal section of the converter, containing a certain amount of water. The domain corresponds to the area $\Omega \subset \mathbb{R}^2$ filled with water at rest (see Fig. 1). Its boundary $\partial\Omega$ is divided in two parts, the converter lower rigid boundary Γ_0 and the liquid surface at rest Γ. Additionally, we consider $\gamma_{\text{in}}, \gamma_{\text{out}} \subseteq \Gamma_0$ the inlet and outlet boundaries, respectively, and d_{in} and d_{out} their diameters.

The fluid is assumed to be inviscid, irrotational and incompressible, then there exists a scalar velocity potential ϕ such that:

$$\Delta\phi(x,t) = 0, \quad x \in \Omega, \quad t > 0. \tag{2.1}$$

On the free surface Γ, the classic approximation of surface gravity waves (cf. [9,10]) is considered. Let η be the free surface elevation relative to equilibrium position. Thus the functions ϕ and η satisfy the next set of relations: