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A spatially dependent model for washing wool

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Abstract

We analytically model the transport of dirt in the industrial washing of wool using the advection–diffusion equation in two dimensions. Separation of variables leads to a Sturm–Liouville problem where the analytic solution reveals how contamination is distributed both along and down the wool and indicates the operating parameter regimes that optimise the cleaning efficiency.

Keywords: Sturm–Liouville; Eigenfunction; Wool scouring

1. Introduction

The industrial process of washing shorn wool is called wool scouring. In an aqueous wool scour machine, wool is fed as a thin layer along the top of bowls of water. Sets of harrows located evenly and almost continuously along the bowls drive the wool forward and agitate it, freeing caught dirt particles. Some dirt is bound to the wool by grease and is removed at the squeeze rollers at the end of the bowl rather than in the bowl itself, but most is loosely bound to the wool and removed relatively easily. Once released, dirt particles settle away from the wool, eventually collecting in tanks at the bottom of the bowl.

A typical scour bowl consists of four tanks. The tanks are periodically drained to remove the dirt, while more water is added, creating a cross-flow along the top of the bowl. A schematic of the conventional scour bowl is shown in Fig. 1.

There have been previous mathematical models of wool scouring that looked at the contaminant movement through the entire scour machine. The simplest model [1] balanced the amount of contamination entering and leaving a scouring machine through a single differential equation to determine the contamination in the entire machine at steady state. Later models [2–6] extended this idea to find the contamination in each of the bowls within a machine and introduced more physical factors. A study group report [7] considered the contaminant concentration within a compartmentalised scour bowl and how it changed with time. This was improved [8] by discretising the scour bowl more finely and using finite differences to get more detailed results. An analytical

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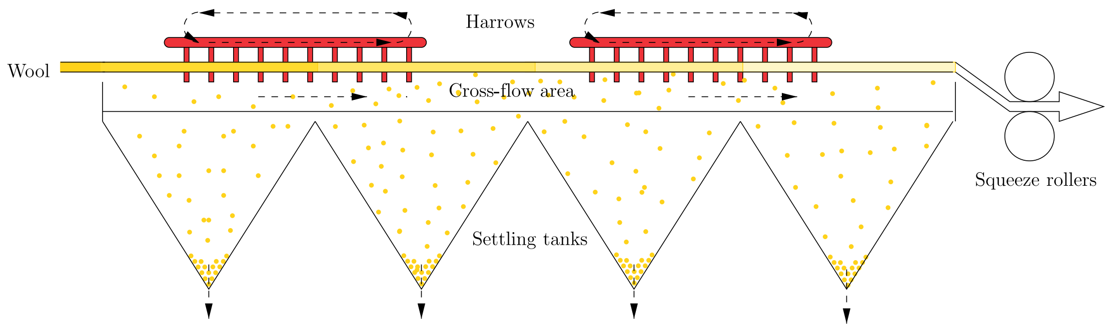

Fig. 1. A schematic of the conventional scour bowl. The harrows move the wool across the top of the water in the bowl and contamination settles out of the wool into the settling tanks. The harrows are shown schematically to indicate their movement, but in reality there are more of them located almost continuously along the bowl.

model of the washing action around the wool at the top of a scour bowl was introduced [9] and some of this work is briefly reproduced in this paper, for clarity and completeness.

The model presented in this paper looks at the concentration of dirt in the wool and water at the top of a single scour bowl. It calculates the steady-state distribution of contamination at the top of the bowl in a two dimensional scenario. In particular, the mixing action of the harrows through and around the wool is taken into consideration. Our aim is to gain a better understanding of how contamination in the wool behaves and see how this model compares with the numerical model of a full scour bowl from [8] and the analytical model of [9]. This model is also a nice example of an application of Sturm–Liouville theory to an industrial problem.

2. Model

This model looks at the top of the scour bowl, where the wool is gently mixed by the harrows. This 'mixing zone' is divided into two layers, the wool and the washing zone, as shown in Fig. 2. The washing zone is where water is mixed with the wool by the harrows. Underneath is the settling zone where dirt moves to the bottom of the tanks under gravity. The movement of contamination in the scour bowl is modelled by the advection–diffusion equation

$$\frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} - v \frac{\partial c}{\partial x} - w \frac{\partial c}{\partial y}, \tag{1}$$

where x is the coordinate down the tank in the direction of settling, y is the coordinate along the tank in the direction of cross-flow, t is time, c is the concentration of contamination at position (x, y), D is a diffusion coefficient which represents the mixing action of the harrows, v is the vertical velocity and w is the horizontal velocity, representing both the wool velocity, w_1 and the cross-flow velocity, w_2.

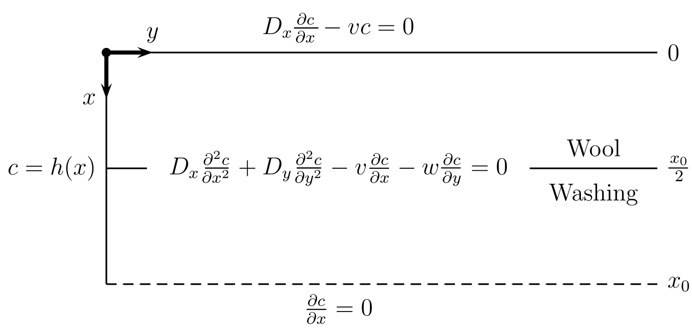

Fig. 2. The mixing zones (wool and washing zone) at the top of the scour bowl with the governing equation and boundary conditions of the model shown.