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Reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli

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Abstract

The model of the equations of generalized magneto-thermoelasticity with two relaxation times in an isotropic elastic medium under the effect of reference temperature on the modulus of elasticity is established. The modulus of elasticity is taken as a linear function of reference temperature. Reflection of magneto-thermoelastic waves under generalized thermoelasticity theory is employed to study the reflection of plane harmonic waves from a semi-infinite elastic solid in a vacuum. The expressions for the reflection coefficients, which are the relations of the amplitudes of the reflected waves to the amplitude of the incident waves, are obtained. Similarly, the reflection coefficients ratios variations with the angle of incident under different conditions are shown graphically. A comparison is made with the results predicted by the coupled theory and with the case where the modulus of elasticity is independent of temperature.

Keywords: Thermal relaxation times; Generalized thermoelasticity theory; Normal mode analysis; Electro-magneto-thermoelastic couple

1. Introduction

In classical theory of thermoelasticity, the thermal waves propagate with an infinite velocity, which is physically impossible. To eliminate the paradox between infinite speed determined by conventional theory and finite speed observed in many experiments for the propagation of thermal signals, some generalized thermoelastic theories were developed. The common used generalized thermoelasticity was proposed by Lord and Shulman [1] and Green and Lindsay [2] (here called LS and GL theories, respectively) by introducing one or two relaxation times in the thermoelastic process. The LS model itself is based on a modified Fourier's law, but the GL model admits second sound even without violating the classical Fourier's law. The two theories are structurally different from one another, and one cannot be obtained as a particular case of the other.

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Nomenclature

$\lambda,\ \mu$	Lame's constants
ρ	density
C_E	specific heat at constant strain
v	Poisson's ratio
t	time
T	absolute temperature
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
k	thermal conductivity
$\mathbf{J}$	current density vector
$\mathbf{H}$	initial uniform magnetic intensity vector
$\mathbf{h}$	induced magnetic field vector
$\mathbf{E}$	induced electric field vector
μ_0	$1/[2(1+v)]$ magnetic permeability
ε_0	electric permeability
δ_0	non-dimensional constant
E_0	constant modulus of elasticity at $\alpha^* = 0$
η_0	$\rho C_E/k$
C_A^2	$\mu_0 H_0^2/\rho$
C_T^2	$(\lambda_0 + 2\mu_0)E_0/\rho$
c^2	$1/\mu_0\varepsilon_0$ sound speed
$v_0,\ \tau_0$	two relaxation times
e	dilatation
α_t	coefficient of linear thermal expansion
γ_0	$\alpha_t/(1+v)$
λ_0	$v/[(1+v)(1-2v)]$
ε^*	$\gamma^2 T_0/(\rho^2 C_E C_T^2)$
β	$E_0(\lambda_0+\mu_0)/(\rho C_T^2)$
R_H	C_A^2/C_T^2
ω_1	$k/(\rho C_E^2 C_T)$

Various problems characterizing the two theories have been investigated, and reveal some interesting phenomena. Brief reviews of this topic have been reported by Chandrasekharaiah [3,4].

The theory of magneto-thermoelasticity is concerned with the interacting effects of applied magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial appliances, particularly in nuclear devices, where there exists a primary magnetic field, various investigation are to be carried out by considering the interaction between magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies. The development of the interaction of electromagnetic field, the thermal field and the elastic field is available in many works such as Abd-Alla [5], Ezzat and Othman [6], Othman [7–10] and Wang et al. [11]. Othman [12–14] used the normal mode analysis and state–space approach to study two-dimensional problems of generalized thermoelasticity with one and two relaxation times with the modulus of elasticity dependent on the reference temperature.

The problem of reflection of plane waves has been discussed by many authors, e.g, Parfitt and Eringen [15], Ariman [16], Singh [17], Singh and Kumar [18,19], Kumar [20], Kumar and Deswal [21] and Song [22] etc. Recently, Othman and Song [23] have studied the effect of rotation on magneto-thermoelastic waves under thermoelasticity without energy dissipation. But few papers concerned with problems of wave propagation