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Magnetic effect on the formation of longitudinal vortices in natural convection flow over a rotating heated flat plate

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Abstract

A numerical study of magnetic effect on the formation of longitudinal vortices in natural convection flow over a rotating heated flat plate is presented. The onset position characterized by the local Grashof number, depends on the rotational Reynolds number, the Prandtl number, the Hartmann number, and the wave number. The Coriolis force and the Lonertz force have significant effects on the formation of longitudinal vortices and the associated instability. Positive rotation stabilizes the flow on the rotating flat surface. On the contrary, a negative rotation destabilizes the flow. The flow is found more stable as the value of Hartmann number increases. The numerical data show reasonable agreement with the experimental results with the case of thermal instability in natural convection over a flat plate heated from below.

Keywords: Natural convection; Grashof number; Longitudinal vortices; Rotation; Magnetic effect

1. Introduction

The problem of the magnetic effect on the thermal instability in a laminar natural convection flow over a rotating heated flat plate has received attention in the heat transfer literature. The instability mechanism is due to the presence of nonlinear interaction of buoyancy force, the Coriolis force, and the Lonertz force. The appearance of longitudinal vortices in a laminar natural convection flow over horizontal plates was observed [1–3]. The experimental and numerical methods employed in the literature on determining the onset of longitudinal vortices were summarized [4–8]. The formations of longitudinal vortices in a rotating boundary layer flow were studied [7–10]. Some laboratory data were used to verify the proposed model.

The study of magneto-hydrodynamics plays an important role in engineering applications, such as harnessing fusion energy, power generation, plasma propulsion, flow control, and metallurgy, etc. The problem of a rotating boundary layer flow under the influence of a magnetic field also has applications in geophysics and astrophysics. Several studies were focused on magnetic effect on the flow and heat transfer behavior in channel flows without rotation (for example, [11–14], etc.). To the author's knowledge, numerical investigation in the

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Nomenclature

a	dimensionless wave number, $a = a'L/Gr_L^{1/5}$
a'	dimensional wave number, $a' = 2\pi/\lambda$ (m^{-1})
B_0	uniform magnetic field strength (Wb/m^2)
F	velocity, pressure or temperature function
f	the reduced stream function, $\psi(v/Gr_X^{1/5})^{-1}$
g	the coefficient of gravity acceleration (m/s^2)
Gr_X	the local Grashof number, $Gr_X = \frac{g\beta(T_w - T_\infty)X^3}{v^2}$
Ha	the Hartmann number, $Ha^2 = \frac{\sigma B_0^2 L^2}{\rho v}$
L	characteristic plate length
p', p	dimensional and dimensionless pressure
Pr	the Prandtl number, v/α
Re_Ω	the rotational Reynolds number, $Re_\Omega = \frac{\Omega L^2}{v}$
T	temperature (K)
t', t	dimensional and dimensionless perturbation temperature, $t' = (T_w - T_\infty)t$
t^0	initial constant perturbation temperature at $x = 0$
U, V, W	dimensional velocity components (m/s)
u, v, w	dimensionless perturbation velocity components
X, Y, Z	Cartesian coordinates (m)
x, y, z	dimensionless Cartesian coordinates

Greek letters

α	thermal diffusivity of fluid (m^2/s)
β	coefficient of thermal expansion (1/K)
δ	boundary layer thickness (m)
η	similarity variable, $YGr_X^{1/5}/X$
θ_b	dimensionless basic temperature, $(T - T_\infty)/(T_w - T_\infty)$
λ	wave length in the Z-direction (m)
ρ	density of fluid (kg/m^3)
τ	the elapsed time of fluid flow from inlet along the streamwise direction
σ	electrical conductivity (mho)
v	kinematic viscosity of fluid (m^2/s)
ξ	vorticity function in X-direction (1/s)
Ω	angular speed (rad/s), defined positive for anticlockwise rotation

Superscript

*	onset position

Subscripts

b	basic flow quantity
p	perturbation quantity
w	wall condition
X	local coordinate
∞	free stream condition

literature of magnetic effect on the longitudinal vortices of natural convection flow in a rotating boundary layer is not yet completely elucidated in the open literature and, thus, it will be analyzed in this paper. The experimental criteria proposed by Hwang and Lin [4] on the onset of longitudinal vortices were employed in the present study. The governing parameters are the Prandtl number, the wave number, the rotational Rey-