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Optimal ordering and pricing policy for an inventory system with trial periods

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ABSTRACT

It is a business practice that home shopping companies offer a free trial period for their products with a goal of increasing sales. Under this policy, if for any reason customers are not satisfied with the purchase, they can return the product for a refund within the trial period. To develop inventory strategies in such environment, home shopping companies should take the return phenomenon into account so as to increase their profit. This paper considers this phenomenon and develops a seasonal inventory model to deal with the problem. Two scenarios are analyzed. In the first scenario, demand is assumed to be linearly price-dependent while in the second one, it is assumed to be exponentially price-dependent. The purpose of this research is to maximize the total profit over a given planning period by determining the optimal ordering quantity and price. The analytical results demonstrate that the optimal ordering quantity and prices are obtained using closed-form formulas.

1. Introduction

Aided by information technology, many companies deliver their product information to consumers at home via websites, e-mails, catalogs, broadcast media, specialty channels on TV, and so forth. For example, sport machine manufacturers advertise their items via sports-related television programs; eBay provides merchandise category over its online stores; and Amazon provides its products information such as books, video games, computer software, electronics and apparel over its website. Home shopping is beneficial to both customers and businesses [1,2]. For customers, it allows customers access to distant stores without involving driving time, have a wider choice in terms of the range of products available, and have convenience in visiting the virtual store at any time. For business, home shopping channels can provide businesses with the ability to release their items to a worldwide market and can reduce their business overhead expenses such as space rental and personnel fee.

Owing to these advantages, more and more enterprises have gone into home shopping businesses. However, in light of these advantages, there are also some weaknesses on home shopping business. It typically does not provide the same levels of product information, personal service, entertainment, and social interaction as do conventional shopping stores. For example, for a virtual electrical appliance retailer, a customer can walk into the electrical appliance retailer, and leave the same day with their purchased electrical appliances. However, for home shopping, a later delivery or some other post-purchase support is required [3]. Examples also include fashion goods, books and so on. For some products such as fashion products, feel and touch are important. However, since customers cannot touch their purchased items before their purchases,

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home shopping is sometimes perceived to be quite risky. For example, the color of the purchased item may not be exactly as it appears when displayed on the computer screen. The mentioned risk is associated with the consumers' belief regarding whether the product would function according to their expectations. In fear of the situation in which the arrived purchased items are not what they ordered, are not what they expected, or do not meet their need, many customers consider home shopping highly risky and have no incentive to make their purchases via home shopping channels.

Being able to target very specific customer groups, home shopping companies have tried to soften customers' risk perception on home shopping. Some home shopping companies offer a free trial period for their products and offers return policies to overcome the above-mentioned drawback. For example, the online shoe retailer Zappos.com does not charge a restocking fee even for returns that are not the result of merchant error. Ulla Popken, a fashion retailer, sells its fashion goods under the condition that customers can return the delivered goods for a full refund or exchange without reason within 14 days of receipt. It is noted that, in home shopping environment with full refund, customers may return their orders after receiving them. However, return of items can be taken as purchase return or order cancellation. Thus, a manager of home shopping business may face a more complicated environment. We can conjecture that without taking this phenomenon into consideration, an inventory decision-maker may over-estimate the actual demand. Thus, from an economic point of view, a home shopping firm must take the phenomenon of purchase return into account when making its inventory and pricing decisions.

The existing related models concentrating on work pertaining to the ordering problem with purchase return are the perishable inventory models with cancellations. We refer the readers to the reviews of McGill and van Ryzin [4], Upchurch et al. [5], Weatherford and Bodily [6] for reference in the field of perishable inventory models.

Burke [2] explored the difficulties to forecast the impact of new communication technologies such as Internet on retailing, discussed the ways how existing retailers respond to this new technology, and suggested how managers might plan for the future. You [7] proposed a dynamic pricing model to deal with a perishable inventory problem under the condition that customer may cancel their order or return purchased goods. Koidea and Ishii [8] dealt with a two-fare hotel overbooking problem with cancellations. You and Wu [9] developed an ordering and pricing policy for an inventory system with order cancellations. Xie and Gerstner [10] discussed an advance selling problem and showed that offering refunds for service cancellations can be profitable. Son [11] dealt with a problem of selecting profitable customer for service industries with two classes of services. They consider the situation in which customer orders may be cancelled due to unavoidable circumstance. Ei-Haber et al. [12] proposed a discrete model to discuss a two-leg airline seat inventory control problem in which the customer behavior such as no-show and cancellation are considered. Xu [13] proposed a multi-period dynamic supply problem in which buyers are allowed to cancel a portion of their outstanding orders with penalty. Fleischmann et al. [14] presented a literature review for inventory policies under consideration of random returns. Kiesmuller et al. [15] dealt with an inventory model in which the random returns depend on the demand stream.

In addition to the above literature, other related models are inventory pricing models. Gupta et al. [16] discussed an inventory model with time- and price-dependent demand. The demand function is assumed to be $D(p) = Ke^{-\beta p}\xi$ where K is a measure of market size, β is a constant value to determine the sensitivity of demand to price p, and ξ is a random variable.

Urban and Baker [17] addressed a deterministic inventory model in which the demand is a multivariate function of price, time and inventory level, and extended the model to a case with a single price markdown. Chun [18] developed the optimal list price and order quantity for seasonal/perishable inventories. The list price is posted at the start of a sales period. Ray et al. [19] dealt with an inventory model with deterministic price-sensitive demand. van den Heuvel [20] dealt with an economic lot-size model deterministic price-sensitive demand. The purpose of their paper is to find an optimal price and ordering decisions simultaneously. You [21] investigated an advance sales system with price-dependent demand. Mandal et al. [22] dealt with the dynamic production inventory problem by utilizing fuzzy time period and inventory costs. Chen and Kang [23] proposed an integrated vendor–buyer model wherein a variant pricing strategy is employed by the vendor to entice its buyer to join a cooperative relationship.

However, the aforementioned literature rarely discussed the inventory problem with trial periods. Without taking into consideration this phenomenon, an inventory decision-maker may over-estimate the actual demand. From an economic point of view, the phenomenon of purchase return cannot be disregarded when making inventory and pricing decisions. In this paper, we aim to develop a strategy for obtaining optimal profit for a firm. According to the proposed strategy, the periodic review pricing policy can be employed to sell seasonal products under the condition that demand for the item is price-dependent and declines with time, and the firm provides the trial period services to customers.

Studies on seasonal inventory models with cancellations or purchase return assume mostly that cancellations may take place over the entire planning period for non-service products or at a certain time point for service products. However, inventory decisions under these assumptions are not necessarily suitable for the situation with trial periods. One of the reasons is that, in many cases, purchase return for a full refund under sales with a trial period is allowed for a limited and specific period. To solve the problem, an inventory model is to be developed by considering this phenomenon. In the present paper, the pricing and ordering decisions are simultaneously determined. Taken together, we develop a seasonal inventory model to resolve the optimal decisions for this problem. Generally, the purpose of this paper is to maximize the total profit through the simultaneous determination of (1) the order quantity and (2) the dynamic sales prices.

The preceding sections of this paper are organized as follows. Section 2 outlines all assumptions made and formulates the problem as a mathematic model. Section 3 analyzes the model in which the demand function is assumed to be a linear function of sales price, and determines the optimal decisions. Section 4 investigated the model in which the demand function is assumed to be an exponential function of sales price, and determines the optimal decisions. Conclusions are finally drawn in Section 5.