Applied Mathematical Modelling 33 (2009) 2927–2933

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analysis of maximum total return in the continuous knapsack problem with fuzzy object weights

Shih-Pin Chen *

Department of Business Administration, National Chung Cheng University, Min-Hsiung, Chia-Yi 621, Taiwan

ARTICLE INFO

Article history:
Received 18 May 2007
Received in revised form 10 September 2008
Accepted 7 October 2008
Available online 17 October 2008

Keywords:
Fuzzy sets
Knapsack problem
Parametric programming
Membership function
Resource allocation

ABSTRACT

This paper proposes a parametric programming approach to analyze the fuzzy maximum total return in the continuous knapsack problem with fuzzy objective weights, in that the membership function of the maximum total return is constructed. The idea is based on Zadeh's extension principle, α-cut representation, and the duality theorem of linear programming. A pair of linear programs parameterized by possibility level α is formulated to calculate the lower and upper bounds of the fuzzy maximum total return at α, through which the membership function of the maximum total return is constructed. To demonstrate the validity of the proposed procedure, an example studied by the previous studies is investigated successfully. Since the fuzzy maximum total return is completely expressed by a membership function rather than by a crisp value reported in previous studies, the fuzziness of object weights is conserved completely, and more information is provided for making decisions in real-world resource allocation applications. The generalization of the proposed approach for other types of knapsack problems is also straightforward.

1. Introduction

The classical knapsack problem involves the situation that a hiker (or a soldier) would like to decide a combination of different objects for his knapsack such that the total value of all objects he chooses is maximized [1]. As generally known, in the literature it is also called the cargo-loading problem, in which a vessel with limited volume or weight capacity is loaded with the most valuable items [2]. In fact, the knapsack model is generalized as a resource allocation model, in that the objective is to maximize the total return under a single resource limitation. Thus it has a variety of applications for capital budgeting, cutting stock, economic planning, and so on (for example, see [3]). Efficient algorithms have been developed for solving knapsack problems when the object weights are known exactly (for example, see [4–12]).

However, there are many cases where the object weights may not be presented in a precise manner. For example, it may be difficult to obtain a reliable estimate of the object weights, particularly when human behavior impacts the operation of the situation. Moreover, as noted in Lin and Yao [13], it may be difficult to determine the precise values of the object weights because the object information is vague. When using decimal truncating functions, dramatic errors may lead to incorrect results. To deal with imprecise information in making decisions, Zadeh [14] introduced the notion of fuzziness.

Clearly, fuzzy input information of this kind will undermine the quality of decisions via conventional crisp knapsack models. Thus knapsack problems with fuzzy object weights deserve further investigation. However, although the knapsack problem with precise object weights has been extensively studied, few research articles have been published on the knapsack problem with fuzzy object weights. Okada and Gen [15] proposed an algorithm for solving fuzzy multiple choice knapsack

* Tel.: +886 5 2720411; fax: +886 5 2720564.
E-mail address: chensp@ccu.edu.tw

0307-904X/$ - see front matter © 2008 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2008.10.004

problems; Kuchta [16] showed the Okada and Gen's algorithm can be applied to a much boarder class of fuzzy multiple choice knapsack problems. Abboud et al. [17] proposed a fuzzy programming approach to multi-objective multidimensional 0–1 knapsack problems. Lin and Yao [13] proposed an approach for constructing fuzzy knapsack models, in that the signed distance ranking method is employed to defuzzify fuzzy object weights. Unfortunately, they only provide crisp solutions with crisp maximal total profits.

Obviously, when the object weights are fuzzy, the maximal total profit will be fuzzy as well. That is, the values of maximum total profit may be different under distinct possibility levels. Therefore, the maximal total profit should be described by a membership function rather than by a crisp value; otherwise, it may lose some useful information. The objective of this paper is to propose an approach with practical merits in case of it is difficult to determine the precise values of the object weights, or dramatic errors may lead to incorrect results when using decimal truncating functions. Aiming at the goal of deriving the membership functions of the continuous knapsack problem with fuzzy objective weights, this paper develops a procedure that is able to provide the membership function of the fuzzy maximal total profit of the fuzzy continuous knapsack problem. The basic idea is to apply a combination of the concept of α-cuts (or α-level sets), Zadeh's extension principle [14,18,19], and the duality theorem of linear programming [2]. A pair of linear programs is formulated to calculate the lower and upper bounds of the α-cut of the fuzzy objective value. The solutions obtained from this pair of linear programs are adopted to construct the membership function of the fuzzy maximal total profit.

In the sections that follow, firstly, the fuzzy continuous knapsack model is briefly stated. Then on the basis of Zadeh's extension principle and the duality theorem of linear programming, a pair of linear programs is developed for calculating the α-cuts of the fuzzy maximal total profit. An example studied by Lin and Yao [13] is solved successfully to illustrate the validity of the proposed method together with some discussions regarding informative benefits of using the proposed approach compared with Lin and Yao's approach. Finally, conclusions are presented.

2. Fuzzy continuous knapsack model

Consider the crisp continuous knapsack problem [13,20], with n objects numbered from 1 to n and a capacity (maximum vessel weight) M. Each object i has a weight $w_i > 0$ and one will earn a profit $p_i x_i$ if a fraction $x_i, 0 \leqslant x_i \leqslant 1$, of object i is placed into the knapsack. Note that we may take fractions of objects into the knapsack. The problem is to determine the optimal combination of objects for the knapsack to satisfy the capacity that maximizes the total profit. Following Lin and Yao [13], the mathematical description of the crisp continuous knapsack problem is

$$\begin{aligned} Z = \max \quad & \sum_{i=1}^{n} p_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i x_i \leqslant M \\ & 0 \leqslant x_i \leqslant 1, \qquad i = 1, 2, \ldots, n. \end{aligned} \tag{1}$$

Intuitively, if the object weights are fuzzy, the maximal total profit Z becomes fuzzy as well. The crisp continuous knapsack problem defined in (1) then turns into the continuous knapsack problem with fuzzy object weights.

Suppose that the object weights $w_i, i = 1, \ldots, n$, are fuzzy because there is only vague knowledge about the objects, and can be represented by the convex fuzzy sets $\widetilde{W}_i, i = 1, \ldots, n$, respectively. Note that a fuzzy set $\widetilde{A}$ is convex in its universal set Y if $\mu_{\widetilde{A}}(\beta y_1 + (1-\beta) y_2) \geqslant \min\{\mu_{\widetilde{A}}(y_1), \mu_{\widetilde{A}}(y_2)\},\ y_1,\ y_2 \in Y,\ \beta \in [0,1]$[20]. Let $\mu_{\widetilde{W}_i}(w_i), i = 1, \ldots, n$, denote their membership functions. Then

$$\widetilde{W}_i = \{(w_i, \mu_{\widetilde{W}_i}(w_i)) | w_i \in S(\widetilde{W}_i)\}, \quad i = 1, \ldots, n,$$

where $S(\widetilde{W}_i)$ are the supports of $\widetilde{W}_i$, which denote the universe set of the weight of object i, $i = 1, \ldots, n$, respectively. The continuous knapsack problem with fuzzy object weights is of the following form:

$$\begin{aligned} \widetilde{Z} = \max & \sum_{i=1}^{n} p_i x_i \\ \text{s.t.} & \sum_{i=1}^{n} \widetilde{W}_i x_i \leqslant M \\ & 0 \leqslant x_i \leqslant 1, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{2}$$

Without loss of generality, all the object weights are assumed to be fuzzy numbers in this model. Note that a fuzzy number $\widetilde{A}$ is a convex and normal fuzzy set [19]. As crisp values can be represented by degenerated membership functions that only have one value in their domains.

Note that the maximal total profit $\widetilde{Z}$ is not a crisp number but a fuzzy number. Consequently, it cannot be obtained directly [21]. We are interested in deriving its membership function $\mu_{\widetilde{Z}}(z)$. In the next section, we will propose a mathematical programming approach based on Zadeh's extension principle for deriving $\mu_{\widetilde{Z}}(z)$.